EXPLICIT COUPLED TDFD TM-LINE DRIVEN BY DOUBLE EXPONENTIAL E-FIELD CONSTANT OVER LINE TO COMPARE WITH SMATH

Reference:C:\Users\Reg\Reg Documents\MATHLIB\SUBROUTINES_MATHCAD\POINTS PER DECADE FUNCTION.xmcd

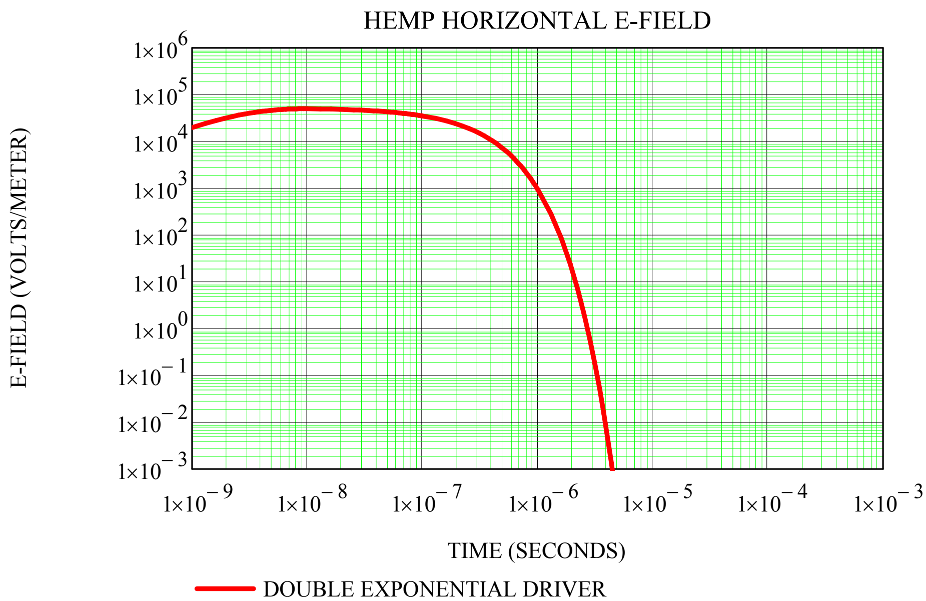
$TSTART := time(1)\cdot sec$

INPUT PARAMETERS FOR PPD SUBROUTINE: $NPD := 30$ $NFS := -9$ $NFF := -1$

$t := PPD(NFS, NFF, NPD)$ $Nt := last(t)$ $Nt = 239$ $k := 0 .. Nt$

$E_o := 5\cdot 10^4$ $\alpha := 4\cdot 10^6$ $\beta := 480\cdot 10^6$ $max(t) = 0.099$

$t_o := \dfrac{\ln\left(\frac{\beta}{\alpha}\right)}{\beta - \alpha}$ $t_o = 1.006\times 10^{-8}$ $Norm := exp(-\alpha\cdot t_o) - exp(-\beta\cdot t_o)$ $E_k := \dfrac{E_o}{Norm}\cdot\left(e^{-\alpha\cdot t_k} - e^{-\beta\cdot t_k}\right)$



$min(E) = 0$

CABLE PARAMETERS FOR CALCULATING COUPLING TO A BURIED CABLE:

Ground Conductivity: $\sigma_g := 1\cdot 10^{-3}$ $\mu_o := 4\cdot\pi\cdot 10^{-7}$

Cable Conductivity: $\sigma_c := 5.8\cdot 10^7$ $c := 3\cdot 10^8$

Ground relative diel const: $\varepsilon_{rg} := 16$ $\varepsilon_o := \dfrac{1}{\mu_o\cdot c^2}$

Cable Conductor Radius $a_o := 10^{-2}$ $\varepsilon_g := \varepsilon_{rg}\cdot\varepsilon_o$

Cable Cell Size & No. of Cells: $Ncell := 100$ $\Delta z := 10$ $Lz := Ncell\cdot\Delta z$ $Lz = 1\times 10^3$

CHANGE ORIGIN OF VECTORS & MATRICES TO COMPARE WITH SMATH

$ORIGIN := 1$

DO TIME ITERATION:

$k := 1 .. Nt - 1$ $\quad Nt = 239$ $\quad \Delta t_k := \mathrm{if}(k \le 1, t_k, t_k - t_{k-1})$ $\quad j := 1 .. Ncell$

DEFINE TRANSMISSION LINE PARAMETERS:

$$Rc_k := \frac{1}{\pi \cdot a_o{}^2 \cdot \sigma_c} \qquad Rs_k := \frac{\mu_o}{4 \cdot \pi \cdot t_k} \qquad R_k := Rc_k + Rs_k \qquad \delta_k := \sqrt{\frac{2 \cdot t_k}{\mu_o \cdot \sigma_g}}$$

$$\delta_k := \left| \begin{array}{ll} \delta_k & \text{if } \ 0 < \delta_k < Lz \\ 1.00001 \cdot a_o & \text{if } \ \delta_k \le 0 \\ Lz & \text{otherwise} \end{array} \right.$$

$$Lg_k := \frac{\mu_o}{2 \cdot \pi} \cdot \ln\left(\frac{\delta_k}{a_o}\right) \qquad L_k := Lg_k \qquad GGg_k := \frac{2 \cdot \pi \cdot \sigma_g}{\ln\left(\frac{\delta_k}{a_o}\right)} \qquad CCg_k := \frac{2 \cdot \pi \cdot \varepsilon_g}{\ln\left(\frac{\delta_k}{a_o}\right)}$$

Make Shorts on Each End of Cable

$$Cg_{j,k} := \left| \begin{array}{ll} 10^{20} & \text{if } \ j = 1 \\ 10^{20} & \text{if } \ j = Ncell \\ CCg_k & \text{otherwise} \end{array} \right. \qquad Gg_{j,k} := \left| \begin{array}{ll} 10^{20} & \text{if } \ j = 1 \\ 10^{20} & \text{if } \ j = Ncell \\ GGg_k & \text{otherwise} \end{array} \right.$$

Approximation to reflection terms by substituting 1/t for iω to get total field

$$R1_k := \frac{2 \cdot \sqrt{\frac{\mu_o \cdot \sigma_g}{t_k}}}{c \cdot \mu_o \cdot \sigma_g + \sqrt{\frac{\mu_o \cdot \sigma_g}{t_k}}} \qquad R2_k := \frac{2\sqrt{\frac{\mu_o}{t_k} \cdot \left(\sigma_g - \frac{\varepsilon_g}{t_k}\right)}}{c \cdot \mu_o \cdot \left(\sigma_g - \frac{\varepsilon_g}{t_k}\right) + \sqrt{\frac{\mu_o}{t_k} \cdot \left(\sigma_g - \frac{\varepsilon_g}{t_k}\right)}}$$

Include Reflection in the driving field: $EE := E$ $\quad EE_k := \mathrm{Re}(EE_k \cdot R2_k)$ $\quad \max(EE) = 2.804 \times 10^4$

Compute Finite Difference Equations:

Decoupled TDFD Calculation

$$II_k := \left| \begin{array}{ll} 0 \ \ \text{if } \ k \le 1 & \\ \dfrac{\dfrac{L_{k-1}}{L_k} \cdot II_{k-1} + \dfrac{\Delta t_k}{L_k} \cdot EE_k}{1 + \dfrac{\Delta t_k}{L_k} \cdot R_k} & \text{otherwise} \end{array} \right.$$

COUPLED TM LINE TDFD ROUTING VS SMATH VERSION

$$
\begin{aligned}
\mathrm{Ans} := \; & \text{for } a \in 1 .. \mathrm{Ncell} \\
& \quad \text{for } b \in 1 .. \mathrm{Nt} \\
& \quad\quad \mathrm{AI}_{a,b} \leftarrow 0 \\
& \quad\quad \mathrm{VG}_{a,b} \leftarrow 0 \\
& \text{for } k \in 2 .. \mathrm{Nt}-1 \\
& \quad \text{for } j \in 1 .. \mathrm{Ncell} \\
& \quad\quad \Delta I \leftarrow \mathrm{AI}_{j,k-1} \quad \text{if } j \le 1 \\
& \quad\quad \Delta I \leftarrow \mathrm{AI}_{j,k-1} - \mathrm{AI}_{j-1,k-1} \quad \text{otherwise} \\
& \quad\quad \mathrm{VG}_{j,k} \leftarrow \frac{\mathrm{VG}_{j,k-1} - \dfrac{\Delta I}{\Delta z} \cdot \dfrac{\Delta t_k}{\mathrm{Cg}_{j,k}}}{1 + \dfrac{\Delta t_k}{\mathrm{Cg}_{j,k}} \cdot \mathrm{GGg}_k} \\
& \quad\quad \mathrm{VT} \leftarrow \mathrm{VG} \\
& \quad \text{for } n \in 1 .. \mathrm{Ncell}-1 \\
& \quad\quad \Delta V \leftarrow \mathrm{VT}_{n+1,k} - \mathrm{VT}_{n,k} \\
& \quad\quad \mathrm{AI}_{n,k} \leftarrow \frac{\mathrm{AI}_{n,k-1} + \dfrac{\Delta t_k}{L_k} \cdot \left(\mathrm{EE}_k - \dfrac{\Delta V}{\Delta z}\right)}{1 + \dfrac{\Delta t_k}{L_k} \cdot R_k} \\
& \quad\quad \mathrm{AI} \\
& \mathrm{stack}(\mathrm{VT}, \mathrm{AI})
\end{aligned}
$$

<----Must initialize inside program!

<----Time Loop on k

<----Spatial Loop on j for Voltage

{----Currents for previous time k-1

{----Voltage at End Point Terminations ZT

{----Voltage at Other Cable Points ≠ ZT

<----Spatial Loop on n for Current

<----Voltages for present time k

<----Current at point n and time k

$V := \mathrm{submatrix}(\mathrm{Ans}, 1, \mathrm{Ncell}, 1, \mathrm{Nt}-1)$

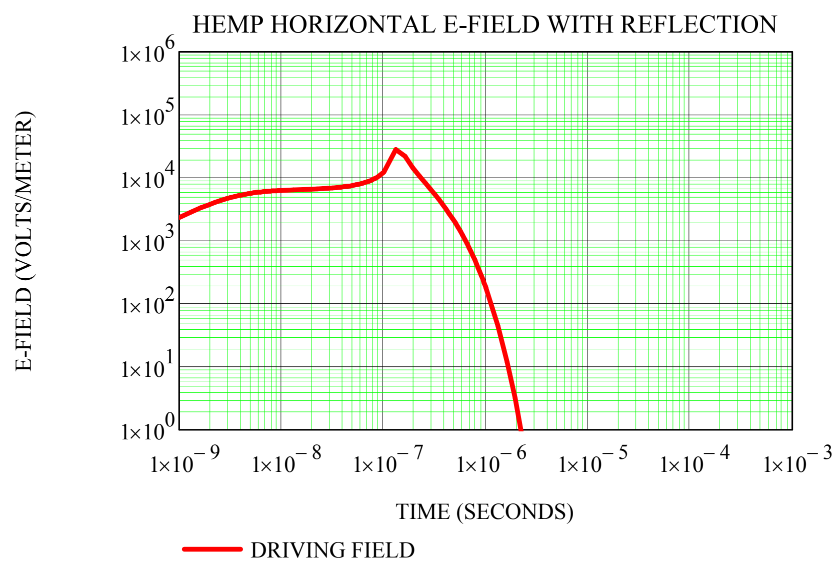
$I := \mathrm{submatrix}(\mathrm{Ans}, \mathrm{Ncell}, 2 \cdot \mathrm{Ncell}, 1, \mathrm{Nt}-1)$

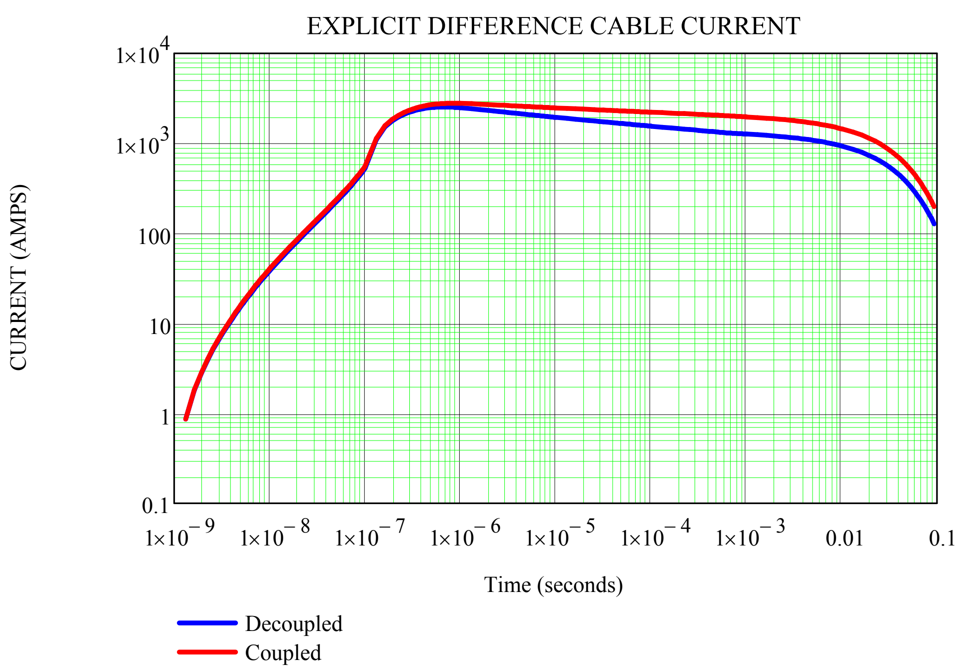
Summary of Answers from Uncoupled and Coupled Models:

Coupled TDFD Results at the center of the transmission line:

$I_{exp_k} := I_{\frac{\mathrm{Ncell}}{2}, k}$ $\quad \max(I_{exp}) = 2.792 \times 10^3$ $\quad V_{exp_k} := V_{\frac{\mathrm{Ncell}}{2}, k}$ $\quad \max(V_{exp}) = 0$

Uncoupled TDFD Result = $\quad \max(II) = 2.536 \times 10^3$





Voltage on cable is 0 at all times & points and the current is spatially constant over the cable as it should be for a spatially constant driving E-field and shorts on both ends.

$$V_{\frac{Ncell}{2},k} = \begin{array}{|c|}\hline 0 \\ \hline 0 \\ \hline 0 \\ \hline 0 \\ \hline 0 \\ \hline \dots \\ \hline \end{array}$$

$I2_k := I_{Ncell,k}$ $\quad \max(I2) = 2.792 \times 10^3$

$I3_k := I_{2,k}$ $\quad \max(I3) = 2.792 \times 10^3$

$I4_k := I_{\frac{Ncell}{4},k}$ $\quad \max(I4) = 2.792 \times 10^3$

CALCULATION OF WAVEFORM NORMS:

$AbsI_k := \left|I_{exp_k}\right|$ $\max\left(I_{exp}\right) = 2.792 \times 10^3$

Rectified Charge (A-sec) === $\sum_k \left(AbsI_k \cdot \Delta t_k\right) = 66.437$ **COULOMBS**

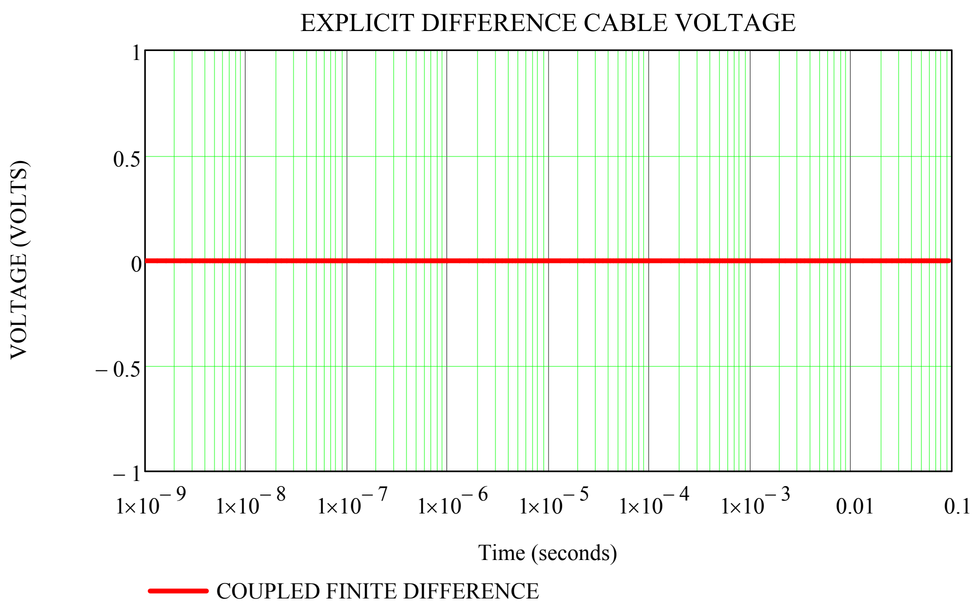
Root Action (A-sec1/2) =====** $ROOT\_ACTION := \sqrt{\sum_k \left[\left(AbsI_k\right)^2 \cdot \Delta t_k\right]} = 260.624$

$k := 1 .. Nt - 2$ $Idot2_k := \left|\dfrac{I_{exp_{k+1}} - I_{exp_k}}{\Delta t_k}\right|$

Maximum dI/dt (A/sec) === $\max(Idot2) = 5.733 \times 10^{11}$

ASSUME 1 OHM LOAD, THE ENERGY DELIVERED TO LOAD IN JOULES= $ROOT\_ACTION^2 \cdot 1 = 6.793 \times 10^4$

68 kilojoules!!!



$time(1) \cdot sec - TSTART = 12.324\,s$