Draghilev's method. Examples.

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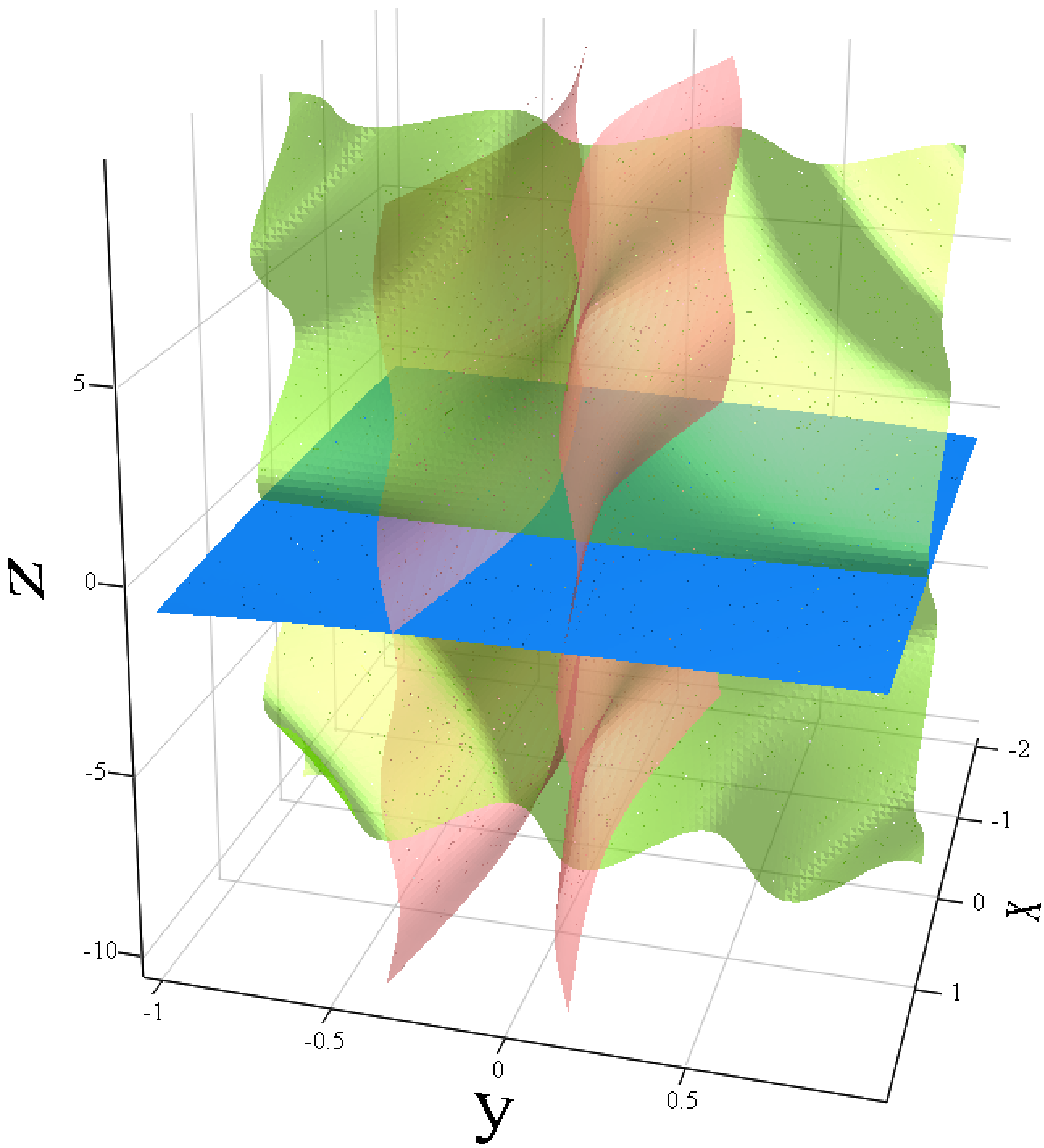
System 3.

$$F(w,y,z) := \begin{pmatrix} 3\cdot w - \cos(y\cdot z) - \frac{1}{2} \\ w^2 - 81\cdot(y+0.1)^2 + \sin(z) + 1.06 \\ 20\cdot z + e^{-w\cdot y} + \frac{1}{3}\cdot(-3 + 10\cdot\pi) \end{pmatrix}$$



A little bit of magic ;)

```
NameVec( n , _v ) := | V := 0
                     | for ii ∈ 1 .. n
                     |   V[ii] := _v[ii]
                     | V
```

$n := 3 \quad X := NameVec(2 \cdot n + 1, x)$

$X1 := submatrix(X, 1, n, 1, 1) \qquad X2 := submatrix(X, n+2, 2 \cdot n+1, 1, 1)$

$X3 := submatrix(X, 1, n+1, 1, 1)$

$X1^T = \begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix}$

$X2^T = \begin{pmatrix} x_5 & x_6 & x_7 \end{pmatrix}$

$X3^T = \begin{pmatrix} x_1 & x_2 & x_3 & x_4 \end{pmatrix}$

```
for ii ∈ 1 .. n + 1
  for jj ∈ 1 .. n + 1
    if ii = jj
      Eorig[ii jj] := 1
    else
      Eorig[ii jj] := 0
```

$$Eorig = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$S := evalm\left(F\left(x_1, x_2, x_3\right) - x_{n+1} \cdot F\left(X_5, X_6, X_7\right)\right)$$

```
for ii ∈ 1 .. n + 1
  | E := Eorig
  | E[n+1 ii] := 1
  | E[ii ii] := 0
  | E[ii n+1] := 1
  | E[n+1 n+1] := 0
  | Y := submatrix( multiply( E , X3 ) , 1 , n , 1 , 1 )
  | if ii = n + 1
  |   out[ii] := maple( |jacobian( convert( S , vector ) , convert( Y , vector ))| )
  | else
  |   out[ii] := maple( -|jacobian( convert( S , vector ) , convert( Y , vector ))| )
  | out
```

$\mathrm{rows}(out) = 4$

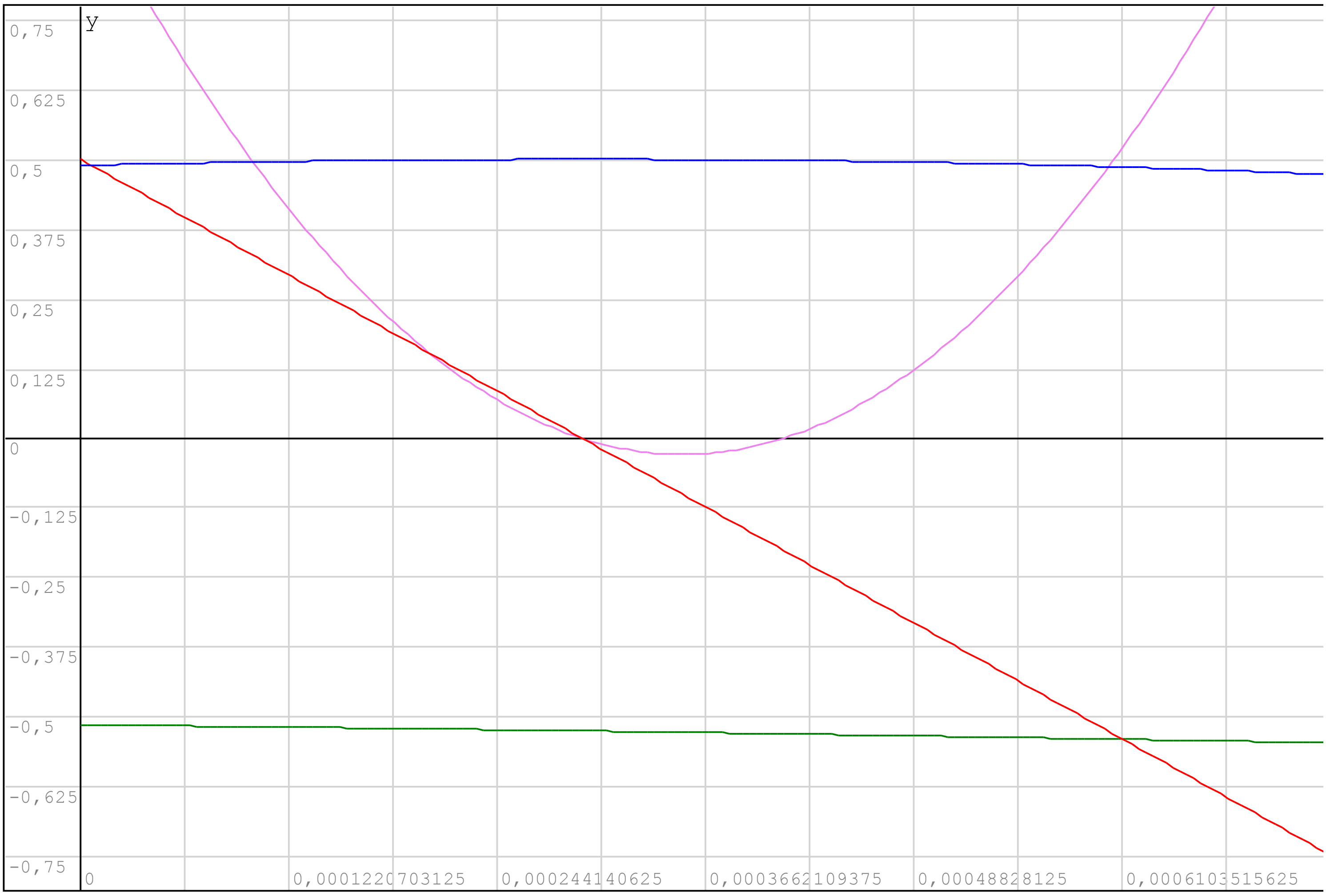
$$dG(x) := \begin{pmatrix} out_1 \\ out_2 \\ out_3 \\ out_4 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$dG(x) = \left( \frac{3 \cdot \left(\left(\left(\left(\left(5 \cdot \left(-\left(4 \cdot \left(-3 \cdot \left(18 \cdot \exp(1)^{x_1 \cdot x_2} \cdot \left(3 \cdot \left(1 + x_2 \cdot \left(2 \cdot \left(5 \cdot \left(1 + 2 \cdot \left(-3 \cdot x_5 + \sin(x_2 \cdot x_3) \cdot x_7 \cdot x_2\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.}{} \right.$$

```
D(t , _x) := dG(_x)

x0 := 0.489    y0 := 0.5    z0 := -0.513    v0 := 1    tmin := 0    tmax := 0.0008

N := 200    X0 := stack(x0 , y0 , z0 , v0 , x0 , y0 , z0)
```

$$dG(X0) = \begin{pmatrix} 73.0321 \\ -1699.673 \\ -29.677 \\ -5828.714 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

```
result := rkfixed(X0 , tmin , tmax , N , D(t , _x))

T := col(result , 1)    X := col(result , 2)    Y := col(result , 3)    Z := col(result , 4)

V := col(result , 5)
```




```
augment(T , X)
augment(T , Y)
augment(T , Z)
augment(T , V)
```

Now we have to find the roots. Each intersection of the parameter V through zero gives us one root.

$out := 0$

```
Search(vector) := | N := length(vector)
                  | k := 1
                  | n := 1
                  | while k < N
                  |   | if (vector_k > 0) ∧ (vector_(k+1) < 0)
                  |   |   | out_n := k
                  |   |   | n := n + 1
                  |   | else
                  |   |   if (vector_k < 0) ∧ (vector_(k+1) > 0)
                  |   |     | out_n := k
                  |   |     | n := n + 1
                  |   |   else
                  |   |     if vector_k = 0
                  |   |       | out_n := k
                  |   |       | n := n + 1
                  |   |     else
                  |   |       n := n
                  |   | k := k + 1
                  | out
```

$Roots := Search(V)$ $\quad rows(Roots) = 2$

A more accurate value of the roots:

$$Interpol(p1, p2, v1, v2) := p1 - \frac{v1}{v2 - v1} \cdot (p2 - p1)$$

```
for ii := 1, ii ≤ length(Roots), ii := ii + 1
  | Rx_ii := Interpol(X_(Roots_ii), X_(Roots_ii + 1), V_(Roots_ii), V_(Roots_ii + 1))
  | Ry_ii := Interpol(Y_(Roots_ii), Y_(Roots_ii + 1), V_(Roots_ii), V_(Roots_ii + 1))
  | Rz_ii := Interpol(Z_(Roots_ii), Z_(Roots_ii + 1), V_(Roots_ii), V_(Roots_ii + 1))
```

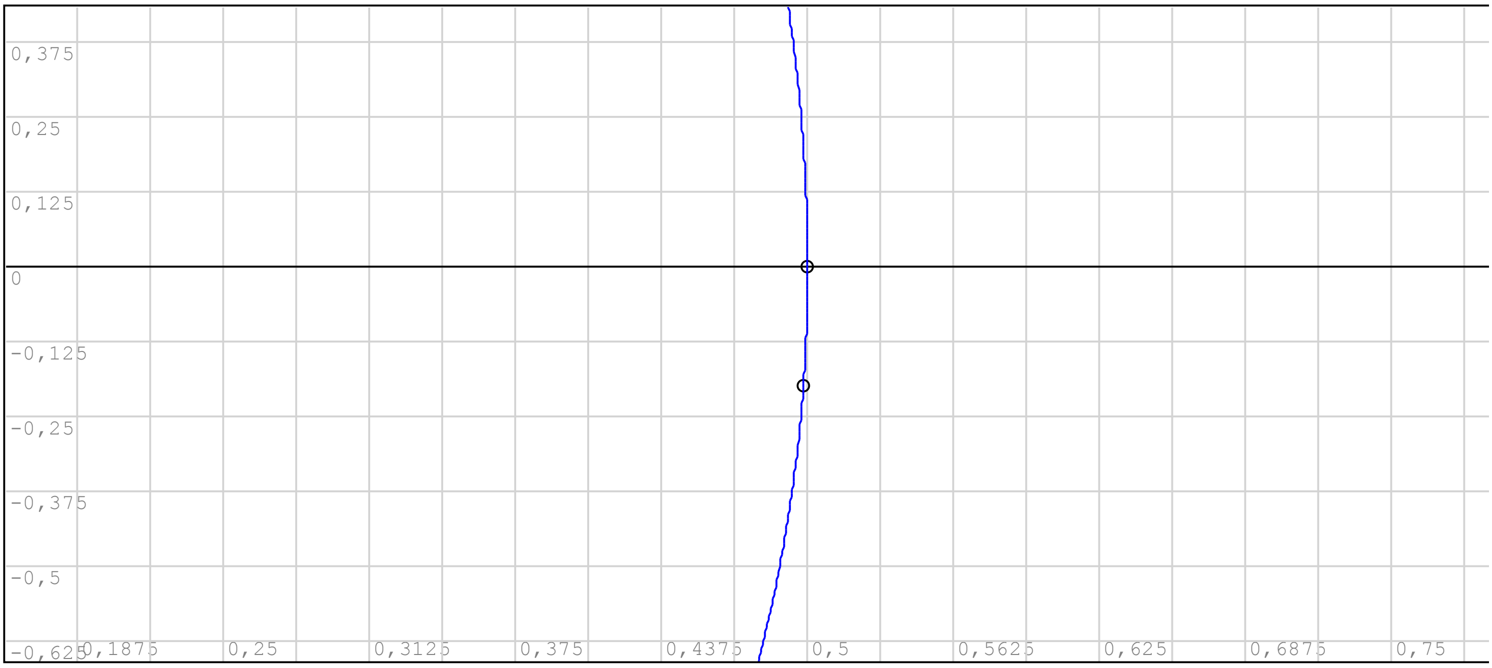
$$Rx = \begin{pmatrix} 0.5 \\ 0.4981 \end{pmatrix} \qquad Ry = \begin{pmatrix} -5.7992 \cdot 10^{-5} \\ -0.1996 \end{pmatrix} \qquad Rz = \begin{pmatrix} -0.5236 \\ -0.5288 \end{pmatrix}$$

$$\frac{1}{\mathrm{rows}(Roots)} \cdot \sum_{ii=1}^{\mathrm{rows}(Roots)} \left| F\left(Rx_{ii}, Ry_{ii}, Rx_{ii}\right)_1 \right| = 0.0003$$

$$\frac{1}{\mathrm{rows}(Roots)} \cdot \sum_{ii=1}^{\mathrm{rows}(Roots)} \left| F\left(Rx_{ii}, Ry_{ii}, Rz_{ii}\right)_2 \right| = 0.0007$$

$$\frac{1}{\mathrm{rows}(Roots)} \cdot \sum_{ii=1}^{\mathrm{rows}(Roots)} \left| F\left(Rx_{ii}, Ry_{ii}, Rz_{ii}\right)_3 \right| = 1.1 \cdot 10^{-6}$$

$\text{for } ii \in 1..\mathrm{rows}(Roc$
$\quad pp_{ii} := \text{"o"}$




$$\begin{cases} \mathrm{augment}(X, Y) \\ \mathrm{augment}(Rx, Ry, pp) \end{cases}$$

$out := 0$

$$V1 := \left| \begin{array}{l} \text{for } ii \in 1..\mathrm{rows}(Roots) \\ \quad out_{ii} := F\left(Rx_{ii}, Ry_{ii}, Rz_{ii}\right)_1 \\ out \end{array} \right.$$

$$V2 := \left| \begin{array}{l} \text{for } ii \in 1..\mathrm{rows}(Roots) \\ \quad out_{ii} := F\left(Rx_{ii}, Ry_{ii}, Rz_{ii}\right)_2 \\ out \end{array} \right.$$

$$V3 := \left| \begin{array}{l} \text{for } ii \in 1..\mathrm{rows}(Roots) \\ \quad out_{ii} := F\left(Rx_{ii}, Ry_{ii}, Rz_{ii}\right)_3 \\ out \end{array} \right.$$

$$Rx = \begin{pmatrix} 0.5 \\ 0.4981 \end{pmatrix} \quad Ry = \begin{pmatrix} -5.7992 \cdot 10^{-5} \\ -0.1996 \end{pmatrix} \quad Rz = \begin{pmatrix} -0.5236 \\ -0.5288 \end{pmatrix}$$

$$V1 = \begin{pmatrix} -1.5863 \cdot 10^{-6} \\ -9.5261 \cdot 10^{-7} \end{pmatrix} \quad V2 = \begin{pmatrix} 0.0009 \\ 0.0005 \end{pmatrix} \quad V3 = \begin{pmatrix} -1.4472 \cdot 10^{-6} \\ -7.5281 \cdot 10^{-7} \end{pmatrix}$$

We can use these initial conditions in order to find more accurate values of the roots.