Big rationals

> bigQ

Introduction

Big integers is a package developed by Overlord, which can do algebra operations with arbitrary precission over integers. A big integer number is a string with only numbers and only one or none minus sign and dot.

$m := \text{"67908374652384958473629504983745832495847238 94"}$

$n := \text{"92948372764503927827304838327"}$

which may or may not have representation in SMath

$a := \text{str2num}(m) = 6.790837465238500 \cdot 10^{45}$

$b := \text{str2num}(n) = 9.294837276450390 \cdot 10^{28}$

Big integer algebra:

$\text{bigMod}(m, n) = \text{"4943526966625997070603214 6345"}$

$\text{bigDiv}(m, n) = \text{"73060315778135387"}$

$\text{bigSub}(m, n) = \text{"67908374652384957544145777338706554222798855 67"}$

bigQ are a sorted couple of big numbers

$r := \begin{bmatrix} \text{"3"} \\ \text{"4"} \end{bmatrix}$ $s := \begin{bmatrix} \text{"12"} \\ \text{"5"} \end{bmatrix}$ $a := \frac{3}{4}$ $b := \frac{12}{5}$

with which you can do algebra and usual calculus, but now with an arbitrary precission, setting the constant 'Digits

$\text{Digits} := 16$

$R := bigQSub(r, s) = \begin{bmatrix} \text{"-33"} \\ \text{"20"} \end{bmatrix}$ $bigQ2D(R) = \text{"-1.650000000000000"}$ $a - b = -1.650000000000000$

$R := bigQMul(r, s) = \begin{bmatrix} \text{"9"} \\ \text{"5"} \end{bmatrix}$ $bigQ2D(R) = \text{"1.800000000000000"}$ $a \cdot b = 1.800000000000000$

$R := bigQDiv(r, s) = \begin{bmatrix} \text{"5"} \\ \text{"16"} \end{bmatrix}$ $bigQ2D(R) = \text{"0.312500000000000"}$ $\frac{a}{b} = 0.312500000000000$

$R := bigQPow(r, s) = \begin{bmatrix} \text{"252427356530549990486293902673644836600884139182398721387140976801386529 8"} \\ \text{"503488304908147320986882340427719104791780456954472170744805181783583900 2"} \end{bmatrix}$

$bigQ2D(R) = \text{"0.501356941302938"}$ $a^{b} = 0.501356941302939$

$bigQ2D(bigQSin(r)) = \text{"0.681638760023334"}$ $\sin(a) = 0.681638760023334$

$bigQ2D(bigQLog(r)) = \text{"-0.287682072451780"}$ $\ln(a) = -0.287682072451781$

$bigQ2D(bigQExp(r)) = \text{"2.117000016612674"}$ $\exp(a) = 2.117000016612670$

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bigQ Math

Doing big integer algebra Digits := 40

Smath expression $N := \frac{\sqrt{3}-1}{2\cdot\sqrt{2}}$

Authomatica conversion to bigQ algebra $R := bigQ2E(N) = bigQDiv\left(bigQAdd\left(\begin{bmatrix}"-1"\\"1"\end{bmatrix}, bigQPow\left(\begin{bmatrix}"3"\\"1"\end{bmatrix}, bigQDiv\left(\begin{bmatrix}"1"\\"1"\end{bmatrix}, \begin{bmatrix}"$

Apply eval $R := \text{eval}(R)$

Big integer $R = \begin{bmatrix}"157056521839177752903510227009491133369240092117876996755798360116\\"606819802526379479300287016906944139497147450559048314194109621079$

Decimal expression

$N = 0.258819045102521$ $D := bigQ2D(R) = "0.258819045102520762348898837624048328349"$

Remember that the actual big rational is R with the couple of integers, and D is only for know what it represent. Some other examples

- $N := \sin\left(\frac{\pi}{12}\right)$ $R := bigQ2E(N)$ $R := \text{eval}(R) = \begin{bmatrix}"5439845892997854995828180582894594\\"210179505563165905421803147791140 98$

  $N = 0.258819045102521$ $D := bigQ2D(R) = "0.258819045102520762348898837624048328349"$

- $N := \frac{1+\sqrt{5}}{2}$ $R := bigQ2E(N)$ $R := \text{eval}(R) = \begin{bmatrix}"652393943937479691747801790669015 73\\"403201631407955854118492697616500 22$

  $N = 1.61803398874989$ $D := bigQ2D(R) = "1.618033988749894848204586834365638117720"$

- $N := 2\cdot\cos\left(\frac{\pi}{5}\right)$ $R := bigQ2E(N)$ $R := \text{eval}(R) = \begin{bmatrix}"523021871795318557495976250321248 42\\"323245293629096854977274781778946 66$

  $N = 1.61803398874989$ $D := bigQ2D(R) = "1.618033988749894848204586834365638117720"$

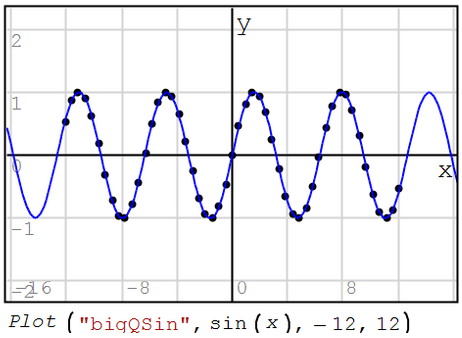
- $N := \ln\left(1+e^{2}\right)$ $R := bigQ2E(N)$ $R := \text{eval}(R) = \begin{bmatrix}"216353086268759246624969982011498 25\\"101720925741472142615593194992138 62$

  $N = 2.12692801104297$ $D := bigQ2D(R) = "2.126928011042972496443726806358304431434"$

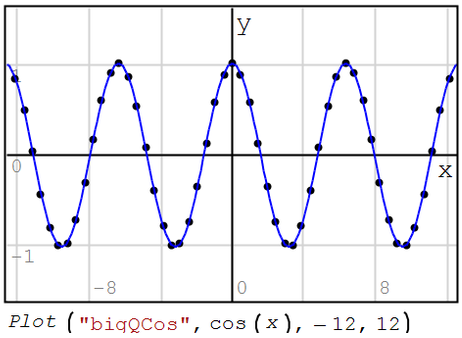
Functions Plots

Digits := 15 $Plot(fs, gs, a, b) := \left\{\begin{matrix} gs \\ \text{augment}\left(X := a + \frac{b-a}{50}\cdot[0..50], \overrightarrow{bigQ2N\left(feval(fs, X)\right)}, "."\right)\end{matrix}\right.$

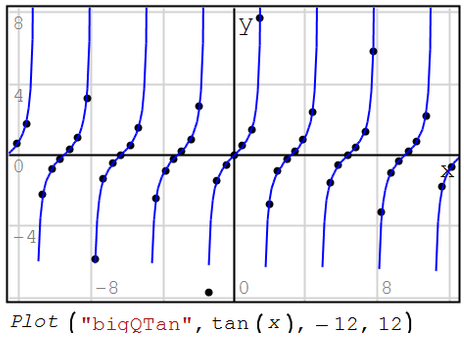
Notice that trigonometric functions Sin, Cos and Tan are well defined in the interval -2π..2π. You can extend outside of that interval using bigQMod.



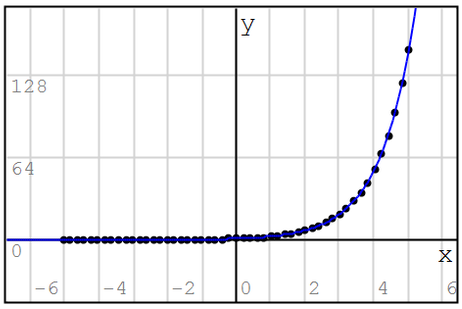
$Plot("bigQSin", \sin(x), -12, 12)$



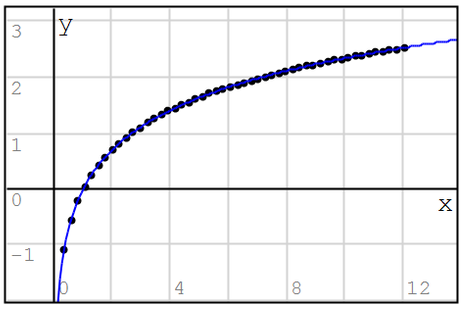
$Plot("bigQCos", \cos(x), -12, 12)$



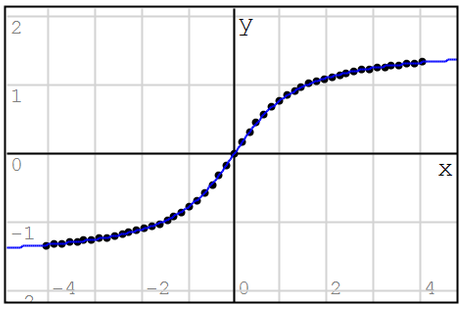
$Plot("bigQTan", \tan(x), -12, 12)$



Plot ("bigQExp", exp(x), −5, 5)



Plot ("bigQLog", ln(x), 0.1, 12)



Plot ("bigQATan", atan(x), −4, 4)

Gallery

Digits := 400

$R_1 := bigQExp(1)$

```
2.71828182845904523536028747135266249775
72470936999595749669676277240766303535475
94571382178525166427427466391932003059921
81741359662904357290033429526059563073813
23286279434907632338298807531952510190115
73834187930702154089149934884167509244761
46066808226480016847741185374234544243710
75390777449920695517027618386062613313845
83000752044933826560297606737113200709328
70912744374704723069697720931014
```

bigQShow (10, bigQ2D (R_1))

$N_2 := 16 \cdot \text{atan}\left(\frac{1}{5}\right) - 4 \cdot \text{atan}\left(\frac{1}{239}\right)$

$R_2 := bigQ2E(N_2)$

```
3.14159265358979323846264338327950288419
71693993751058209749445923078164062862089
98628034825342117067982148086513282306647
09384460955058223172535940812848111745028
41027019385211055596446229489549303819644
28810975665933446128475648233786783165271
20190914564856692346034861045432664821339
36072602491412737245870066063155881748815
20920962829254091715364367892590360011330
53054882046652138414695194151160
```

bigQShow (10, bigQ2D (R_1))

bigQShow (10, bigQ2D (R_2))

Continued fractions

$a2(n, x) := ["1" \ "1"]$

$b2(n, x) := \begin{cases} ["1" \ "1"] & \text{if } n = 0 \\ ["2" \ "1"] & \text{otherwise} \end{cases}$

$R_1 := bigQCF("a2", "b2", 0)$

```
1.41421356237309504880168872420969807856
96718753769480731766797379907324784621070
38850387534327641572735013846230912297024
92483605585073721264412149709993583141322
26659275055925597557999505011527820605714
70109559971605970274534596862014728517418
64088919860955232923048430871432145083976
26036279952514079896872533965463318082964
06206152583523950547457502877599617298355
75220337531857011354374603408498841
```

bigQShow (10, bigQ2D (R_1))

$a\Pi(n, x) := \begin{cases} bigQ(4) & \text{if } n = 1 \\ bigQ\left((n-1)^2\right) & \text{otherwise} \end{cases}$

$b\Pi(n, x) := \begin{cases} bigQ(0) & \text{if } n = 0 \\ bigQ(2 \cdot n - 1) & \text{otherwise} \end{cases}$

$R_2 := bigQCF(a\Pi, b\Pi, x)$

```
3.14159265358979323846264338327950288419
71693993751058209749445923078164062862089
98628034825342117067982148086513282306647
09384460955058223172535940812848111745028
41027019385211055596446229489549303819644
28810975665933446128475648233786783165271
20190914564856692346034861045432664821339
36072602491412737245870066063155881748815
20920962829254091715364367892590360011330
53054882046652138414695194151160
```

bigQShow (10, bigQ2D (R_2))

bigQ Newton Raphson

Nowton-Raphson Method

$\text{Clear}(f, x) = 1$

$\text{Digits} := 50$

$f(x) := x^2 - \ln(3)$

$xo := 3$

$ro := bigQNR(f, xo)$

$\sqrt{\ln(3)} = 1.04814707396821$

$bigQ2D(bigQ2E(ro)) = \text{"1.0481470739682049464912375033233650158031922404764"}$

$bigQ2D\left(bigQ2E\left(f(x)\Big|_{ro}\right)\right) = \text{"-0.0000000000000000000000000000000000000000000000000000"}$

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bigQ RREF

Row reduced echelon form

$\text{Digits} := 15$

$$M := \text{augment}\left(\begin{bmatrix} 3.5 & -2.9 & 1.4 \\ 6.7 & 3.9 & 8.4 \\ 4.8 & -8.1 & 4.2 \end{bmatrix}, \text{identity}(3)\right)$$

$$Mr := \overrightarrow{bigQ(M)} = \begin{bmatrix} \begin{bmatrix}"7"\\"2"\end{bmatrix} & \begin{bmatrix}"-29"\\"10"\end{bmatrix} & \begin{bmatrix}"7"\\"5"\end{bmatrix} & \begin{bmatrix}"1"\\"1"\end{bmatrix} & \begin{bmatrix}"0"\\"1"\end{bmatrix} & \begin{bmatrix}"0"\\"1"\end{bmatrix} \\ \begin{bmatrix}"67"\\"10"\end{bmatrix} & \begin{bmatrix}"39"\\"10"\end{bmatrix} & \begin{bmatrix}"42"\\"5"\end{bmatrix} & \begin{bmatrix}"0"\\"1"\end{bmatrix} & \begin{bmatrix}"1"\\"1"\end{bmatrix} & \begin{bmatrix}"0"\\"1"\end{bmatrix} \\ \begin{bmatrix}"24"\\"5"\end{bmatrix} & \begin{bmatrix}"-81"\\"10"\end{bmatrix} & \begin{bmatrix}"21"\\"5"\end{bmatrix} & \begin{bmatrix}"0"\\"1"\end{bmatrix} & \begin{bmatrix}"0"\\"1"\end{bmatrix} & \begin{bmatrix}"1"\\"1"\end{bmatrix} \end{bmatrix}$$

$$Mo := bigQRREF(Mr) = \begin{bmatrix} \begin{bmatrix}"1"\\"1"\end{bmatrix} & \begin{bmatrix}"0"\\"1"\end{bmatrix} & \begin{bmatrix}"0"\\"1"\end{bmatrix} & \begin{bmatrix}"23271780"\\"43544858"\end{bmatrix} & \begin{bmatrix}"33080"\\"6220694"\end{bmatrix} & \begin{bmatrix}"-710"\\"3761"\end{bmatrix} \\ \begin{bmatrix}"0"\\"1"\end{bmatrix} & \begin{bmatrix}"1"\\"1"\end{bmatrix} & \begin{bmatrix}"0"\\"1"\end{bmatrix} & \begin{bmatrix}"479660"\\"6220694"\end{bmatrix} & \begin{bmatrix}"314260"\\"6220694"\end{bmatrix} & \begin{bmatrix}"-1430"\\"11283"\end{bmatrix} \\ \begin{bmatrix}"0"\\"1"\end{bmatrix} & \begin{bmatrix}"0"\\"1"\end{bmatrix} & \begin{bmatrix}"1"\\"1"\end{bmatrix} & \begin{bmatrix}"-12165"\\"26327"\end{bmatrix} & \begin{bmatrix}"2405"\\"26327"\end{bmatrix} & \begin{bmatrix}"16540"\\"78981"\end{bmatrix} \end{bmatrix}$$

$$\overrightarrow{bigQ2N(Mo)} = \begin{bmatrix} 1 & 0 & 0 & 0.534432331826642 & 0.005317734645041 & -0.188779579898963 \\ 0 & 1 & 0 & 0.077107152353098 & 0.050518479127892 & -0.126739342373482 \\ 0 & 0 & 1 & -0.462073156835188 & 0.091351084438029 & 0.209417454830909 \end{bmatrix}$$

$$M_{[1..3][1..3]}^{-1} = \begin{bmatrix} 0.534432331826642 & 0.005317734645041 & -0.188779579898963 \\ 0.077107152353098 & 0.050518479127892 & -0.126739342373482 \\ -0.462073156835188 & 0.091351084438029 & 0.209417454830909 \end{bmatrix}$$

Alvaro