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Improving the learning of thickening design through graphical methods with the freeware software SMath studio

Juan J. Gallardo-Rodríguez[1,2] | Ana Ruiz-Ortega[3] | Elvira Navarro-López[1] | María del Carmen Cerón-García[1] | Alba Beas-Catena[1,2,3] | Asterio Sánchez-Mirón[1] | Francisco García-Camacho[1]

[1]Department of Chemical Engineering, Higher Engineering School, University of Almería, Almería, Spain

[2]Department of Chemical Engineering, Faculty of Engineering, University Concepción, Concepción, Chile

[3]Department of Humanities, Faculty of Education and Social Sciences, Andrés Bello University, Concepción, Chile

Correspondence
Juan J. Gallardo-Rodríguez, Department of Chemical Engineering, Higher Engineering School, University of Almería, Almería 04120, Spain.
Email: jgr285@ual.es

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Abstract
Thickening consists of separating parts of a suspension by sedimentation under the force of gravity, to obtain a denser product and a clarified liquid. Depending on which part is needed to recover, the denser product or the pure liquid, we use the term thickening or clarification to define these operations. Thickener design is compulsory for chemical engineering (CE) curricula and is usually introduced in courses related to unit operations. Thickening design problems are solved by geometric methods (e.g., Coe–Clevenger or Kynch–Roberts methods). However, nowadays, computers play a more and more important role in CE practice. Both specific and general software solutions are essential. Whereas CE-specific software is adequate for graduates and professionals, general mathematical software can also be used in class. They can help to deliver certain competences included in the vast majority of CE curricula. For CE introductory courses, software with integrated functions and friendly interfaces are generally preferred over those that require to be familiarized with programming languages. Here, we describe the use of SMath Studio in 2nd-year engineering courses by nonfamiliarized students. SMath Studio is a comprehensive mathematical notebook, similar to Mathcad, but it is entirely free for personal or commercial use. In this study, student's opinion was surveyed. Around 70 students took part in the two-phase survey. The use of SMath allowed students to better understand the theoretical basis, fostered critical thinking, and increased their motivation to apply this software for solving other engineering problems.

KEYWORDS
freeware, graphical methods, thickeners, unit operations

1 | INTRODUCTION

Progress in chemical engineering (CE) is achieved through the development of nano- or microscale models whose complexity will evolve alongside the computational capabilities available in the future. In chemical engineering education (CEE), as computers play an increasingly more important role in engineering practice, the specific weight of certain CEE competences should be taken into account and fostered.

The use of mathematical software packages to solve engineering problems was first introduced in the early 1980s [1], mainly in the industry [14]. Their use allows professionals to significantly reduce the time and effort required in engineering calculations [10,12]. Since the 1980s, they have become increasingly more popular [2,10] and, consequently, they are being integrated in engineering curricula worldwide [10]. When properly integrated, this software can provide significant benefits to the overall learning process [14]. Computer-assisted learning has become an essential tool to consolidate theoretical knowledge [6]. Learning through simulation is very effective, because students are able to focus on the problem analysis, test the influence of several parameters, develop and compare alternative solutions, and verify and validate these solutions [2,14]. Students only need to input the mathematical model and the data, and the program will carry out all the steps necessary to reach the solution to the problem presented. This, among other advantages, has made its use preferable than the use of calculators and/or spreadsheets [13].

Mathematical packages must be highly accurate, easily adapted and modified, highly efficient with regard to problem-solving, and cost-effective; besides, they should also be user-friendly, simple and versatile, with a short learning curve, and should require minimal user intervention [10,13,14]. There are, however, some disadvantages of the use of mathematical software. For example, students need to be able to use their judgment to determine if a solution is valid and makes sense. Trusting the software to come up with the right solution, as well as trying to solve a problem through the trial and error method, instead of using engineering analysis, might prevent them from doing so [14]. Some mathematical software require training in programming [10], and modeling of some real-life situations can be very complex to engineering students [6]. Therefore, professional software solutions are not the most suitable for students from a pedagogical point of view. This is especially relevant for first-year courses, as difficulty in the use of software could hinder the learning of basic principles of the discipline.

The most widely used packages are Mathcad, Matlab, and Excel, mostly due to their accessibility [5]. In education, optimization problems can be solved using spreadsheets, such as Excel, especially if the problem involves a small number of equations and/or variables. This is not the case in the industry, in which problems are much more complex and require a more powerful computing capacity [6]. Shacham and Cutlip [13] compared six different mathematical packages to solve typical benchmark engineering problems: Polymath, Mathcad, Mathematica, MatLab, Maple, and Excel. They found that all six packages performed adequately regarding numerical performance, but their user-friendliness and technical effort requirements varied significantly. Mathematica and Maple require extensive technical effort and knowledge of the syntax, whereas using Excel involves extensive programming with Visual Basic, unless the problem is relatively simple (e.g., simple variable iterative processes). However, valuable pedagogical approaches in CE can be found with these packages (for instance, [1]).

Software with integrated functions and a friendly interface are preferred to those where a knowledge of programming languages is essential. Thus, in CEE, the two most widely used ones have been MathCad and Matlab. Matlab is a hybrid source iterative computer software used for solving problems or evaluating systems, which enables users to conduct mathematical operations of any kind [5]. Matlab also incorporates Simulink, a block diagram environment for multi-domain simulation and model-based design. However, to solve problems in Matlab, students must be familiar with MatLab syntax, as well as they must be able to write programs, so that Matlab carries out all the calculations and shows the solution. Most users have little to no knowledge of how to program in Matlab, which makes an introduction to basic commands a necessity [10,13]. Many examples of Matlab use as a pedagogical tool can be found in literature [7,10,11]. Mathcad is an interactive hybrid software used to perform mathematical operations and is compatible with most software systems [5]. It is a command-based software, and the user must specify which integration algorithm to use and the number of data points to be plotted [13]. In this sense, MathCad presents a favorable learning curve, which allows to solve relatively complex systems in the classroom without previous knowledge of the software [5]. As in other application areas, free software solutions have been made available in the last years. SMath Studio is a free mathematical software that is very similar to Mathcad in terms of use and application. Although its calculation capacity is much smaller than that of Mathcad, it can be used successfully in engineering calculations [15]. It was developed specifically for engineering purposes as a free-to-use software. It has a user-friendly graphical interface, which is connected to a powerful math engine core, and it can, among others, perform numeric and symbolic calculations, plot 2D and 3D graphs, perform operations with units, has the capacity to use programming functions, and so forth. SMath also comprises a cloud-based productivity tool, SMath Live, which ensures quick and easy access to data from a variety of devices [8].

1.1 | Unit operations' teaching

Graphical methods in CEE have been used for decades, as they facilitate the calculus of complex systems. Typically, plot equilibrium data and mass/energy balances are easily solved using an algorithm to determine the design parameters. In the pre-computer era, these methods were used by professional engineers, academics, and students. Nowadays, they are almost exclusively employed as a learning methodology in class. CE software (e.g., SuperPro, Aspen Hysys) are not used in introductory courses, as the theoretical basis and calculus algorithm are hidden; thus, the pedagogical function is limited at this stage. Besides, learning by heart the steps without a deep understanding is still an option for students to prepare for exams. A compromise could be the use of a generic mathematical software with plotting functions. For the solving sequence to be programmed, more attention should be paid to the algorithm to avoid mistakes. The fact that the students can "see" at the same time the effect of the variables on the results can help to improve their understanding of the subject, thus promoting critical thinking. In addition, programming and symbolic mathematics can be used to design scenarios where students must develop higher order cognitive skills.

In thickening operation design, different graphical methods have been traditionally used to determine a critical area design. The design variables depend on a complex relationship between the sedimentation velocity and sludge concentration. Therefore, it poses an interesting design problem where an underlying theory and the practical engineering approaches can be taught by means of examples solved with mathematical programs. In this study, we present a methodology for teaching thickener design using the free software SMath Studio, where autonomous learning and critical thinking have been specially fostered.

2 | TEACHING METHODOLOGY

2.1 | Background

This teaching strategy has been applied in the Bioengineering degree offered by the Faculty of Biological Sciences of the University of Concepción (Chile). Four mandatory courses are delivered by the Department of Chemical Engineering. The degree syllabus highlights the competence in carrying out basic and applied research, and applying this knowledge to the industrial field. The educational model is based on learning outcomes and it is focused on the development of competencies (generic and specific). The continuous education offered by the University converges with international trends, specifically with the Guidelines of the European Higher Education Space after the Bologna process. Specifically, the methodology described here was used in the "Unit operations in Bioengineering I" course. This course intends to provide the basic foundations of engineering sciences, especially fluid mechanics and heat transfer, and their relationship with the associated unit operations that are presented in the processing of biomaterials. It has four credits (total of 40 hr) and is delivered in the 2nd year of a 6-year curriculum. It adheres to the following competencies of the syllabus: ability to understand, solve, and suggest solutions to problems where there exists fluid and heat transfer in the bioprocess industry. The expected learning outcomes are as follows: (a) identifying the main units of measurement and physical properties related to fluid behavior and heat transfer; (b) describing the variables used in mass and energy balances calculations, applied to fluid and heat transfer; (c) understanding and applying the fundamental principles of static and fluid dynamics, and their interaction with equipment in biological/bioprocess systems; (d) understanding and applying the fundamental principles of heat transfer in biological/bioprocess systems to problems; (e) understanding the main equipment and engineering operations involved in fluid mechanics and heat transfer; and (f) recommendation of separation, mixing, fluid pumping, and heat transfer according to the plant's operating needs.

2.2 | Course general methodology

The general teaching methodology used was based on the application of project-led engineering strategies. The number of students was 20–25 per semester. The course evaluation was done through theoretical tests and a final practice test. Theoretical tests consisted of seven-question written assays that sought to verify the level of understanding of fundamental knowledge. In total, six theoretical tests were carried out, one for each topic. The practice test consisted of two design problems that the students had to solve using the mathematical software SMath Studio. Design problems simply consisted of the preliminary design of the different unit operations included in the curriculum of the course (e.g., fluid pumping, heat exchange, centrifugation, etc.).

2.3 | Thickeners' design teaching methodology

For this unit, the methodology was designed to reach the objectives included in Table 1. The module was delivered

TABLE 1 Objectives of the teaching methodology

Students should be able to introduce numeric variables with units, produce plots, and use Boolean operators in SMath
Students should be able to use plugins with mathematical methods, routines, and specific functions, which can be downloaded for integration in the software. Specifically, polynomic regression must be studied
Students should be able to calculate a critical thickener area from a data set using different methods
Students should be able to discuss the effect of different flows and required underflow concentration on the design area
Student should understand the limitations of the underlying theory and the engineering approach

in four two-hour sessions divided in two parts. Thus, 8 hr of in-class activities were needed (out of a total of 40 hr). Two types of classes were delivered: standard lectures and seminar classes.

The first part of the teaching module was dedicated to the software. A 2-hr class was delivered to introduce the program. In this class, introduction of numeric variables (directly and using the Excel import function), the use of units, plots, and the fundamentals of Boolean operators were presented and practiced. Several examples were given to the students to be solved at home. A second seminar class was designed to foster autonomous learning. Students were told to review the SMath web and forum, and search for "plugins" to integrate new functions into SMath. They were required to fit a data set with polynomic regression. In the second part, a lecture was delivered focusing on the thickening operation with an historic perspective and the different methods (see Section 2.4) that have been used so far. These contents have been included in the Supporting Information Material. Examples of design procedures were presented in class. The last seminar consisted of the resolution of a design problem using SMath in groups of two or three students. The problem is presented below (Section 2.5). The expected outcome is also appended below (Figures 2–5).

2.4 | Methods for thickening design

Thickening consists of the separation of a suspension by gravitational sedimentation to obtain a denser product (slurry) and a clarified liquid. Depending on which stream needs to be recovered, the process is called either thickening or clarification, respectively. Industrially, this process is performed in a thickener. Metallurgical and chemical plants at the beginning of 20th century already had thickeners, which were very similar to the current industrial equipment (see Figure 1). As the feed stream is usually known, the main parameters to be determined are the area and the height of the thickener. Typical solid flux values for different types of streams range between 0.8 and 5 kg solids/hr/m^2 [4]. For mechanically agitated thickeners, the area and height values usually range between 10 and 100 m^2 and 2.5 and 3.5 m, respectively. The area is determined using the relationship between settling velocity and concentration, obtained from batch settling experiments, even though they do not simulate accurately the flux patterns of a continuous unit. Historically, and especially in the mining industry, the

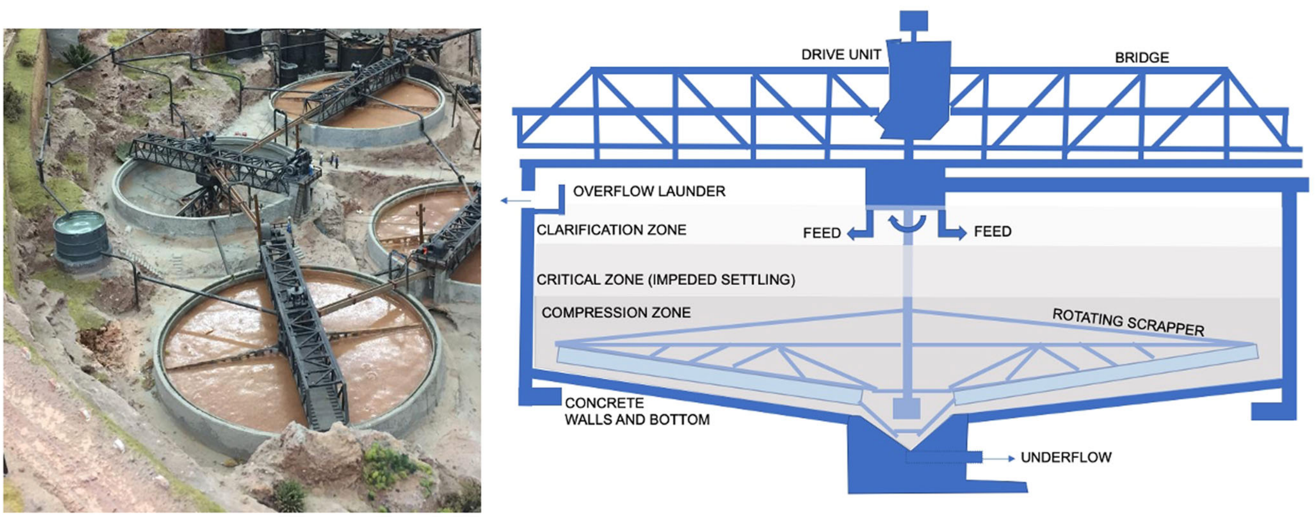


FIGURE 1 The model of mining thickeners (left) and schematic view of a bridge-type thickener (right). Fluid zones have been not-to-scale drawn

TABLE 2 Methods for estimation of the critical thickener area in continuous systems from batch settling tests

Method	Required plot	Area equation
Coe–Clevenger[a]	h-t	$A = \frac{L_0 \cdot C_0}{V_C}\left(\frac{1}{C} - \frac{1}{C_u}\right)$
Total solid flux method[a]	S_T -C	$A = \frac{L_0 \cdot C_0}{(V_C + V_t) \cdot C}$
Yoshioka[a]	S_C -C	$A = \frac{L_0 \cdot C_0}{S_C}$
Talmage–Fitch	h-t	$A = \frac{t_u}{h_0} L_0$
Merta–Ziolo	h-t	$A = \frac{L_0}{h_0} \cdot t_n$

[a]In these cases, an area is obtained for each pair of V_C-C. The highest value must be selected. V_C is the settling velocity at a given concentration C in a discontinuous system and V_t is the transport velocity of the solids due to fluid movement. C_0 and C_u are the initial and underflow concentration, respectively. L_0 represents the volumetric feed flow. S_C is settling flux ($V_C \times C$). S_T is the total solid flux (settling and transport). t_u and t_n are critical times. h_0 is the initial height in an h-t plot from a batch settling test.

Coe–Clevenger method has been the preferred method used by manufacturers to calculate the area of a thickener [3]. However, a security factor must be used, as the method typically underestimates the design area (specially for concentrated underflows; see Supporting Information Material). There are other methods that can be used to determine the critical area of a thickener. The main difference between these methods and the Coe–Clevenger method is that the latter requires various batch settling tests at different concentrations. According to the Kynch theory, in a single batch settling test, solids will pass through all attainable concentrations; thus, settling velocities for each one can be obtained [9]. As a summary, the design equations applied are shown in Table 2. The steps for this method and the calculus algorithms are described in Supporting Information Material.

2.5 | Determining the area of the thickener

Students were asked to determine the area of a thickener to treat a stream with a feed flow rate (L_0) of 100 m^3/hr, initial solids' concentration (C_0) of 25 kg/m^3, and a slurry outlet flow (G_n) of 0.007 $m^3 \cdot m^{-2} \cdot min^{-1}$. A discontinuous batch settling test of a graphite solution in toluene was performed and the h-t data were provided (data were obtained from [9]). The resolution was a seminar activity and students were asked to register how much time they used. In this case, the underflow concentration, C_u, is not known, but the slurry outlet flow has been imposed.

2.6 | Evaluation and surveys

Several methods were used to evaluate the student's learning on this topic. In addition to the quizzes to probe and assess their level of comprehension and growth in understanding of the theoretical basis, the following activities were also conducted: We used the Socratic method during the seminar classes. A set of probing questions and "what ifs" about the exercises were asked to verify the level of understanding of the students. The students were provided with a series of open-ended questions to be answered as a part of their lab report. These questions involved interpretation, bibliographic search, data estimation, and decision-making.

To determine how the students evaluated the experience of using mathematical software in engineering problems, a survey was conducted at the beginning and at the end of the course. The total number of students who took part in this survey was 70. At the beginning of the course, students were asked about their previous knowledge of SMath or any other mathematical software. This was done to adapt the existing teaching methodology to their previous knowledge of this software. At the end of the course, students were asked (8-question survey) about their experience using SMath to calculate the area of a thickener, instead of the traditional way, to teach and solve these engineering problems. The questions asked are shown in Table 4.

3 | RESULTS AND DISCUSSION

One key aspect in the learning process is the motivation to learn. University students also have a critical view of the teaching methodologies. Before the beginning of the course, students were asked about their previous knowledge of SMath or other mathematical software to solve engineering problems. As per the answers, only 10% of the students were already familiarized with the use of SMath, whereas at least 7% of the students admitted knowing about other mathematical software, such as Mathcad or MatLab, to solve similar problems. The rest, 83%, had never used SMath or any other similar software before. As the majority of the students were not familiar with the use of mathematical software to solve engineering problems, an introductory lesson was necessary to give them some basic notions and help them understand how it works, and thus to improve their understanding of the theoretical and practical knowledge of the subject. If only a small percentage of students have any prior programming knowledge, this certainly poses an important issue when introducing mathematical software in engineering.

TABLE 3 Examples of questions used to evaluate the students' learning and foster critical thinking

Class	Socratic questions examples	Open-ended questions examples
SMath introduction	What if a mathematical operation could not be performed with units (e.g., log(x))?	How can the calculation sequence for a list of values in SMath be repeated?
	What if a variable is defined twice in the document?	
SMath advanced	How can we know that the derivative of a function is the equation of the function's tangent?[a]	How can we fit data to a curve in SMath?
Lecture on thickening	What would be the length required for a particle to settle down if it has given an initial velocity of 2 m/s and a settling velocity of X m/s	How can the effect of pressure be accounted for in the thickening operation?
	What if the settling velocity is reduced?	
Thickener design using SMath	Why are different area values obtained if the underlying theory is shared?	How can you obtain the intercept of a tangent function to the G-C plot that crosses the x axis at the maximum attainable concentration in SMath?[b]

[a]Necessary for the calculation of the settling velocity from a batch data.
[b]Necessary for the Yoshioka method (see Figure 5).

However, SMath Studio was shown to have a quite short learning curve. Four contact hours were enough for students to be familiar with the software and solve simple examples. During the second class, where students were asked to do some research on how to implement plugins in SMath, there were many questions regarding the concepts covered in the first class. The immediate application of the software to obtain the required polynomic fitting was key to have a participative attitude in class. The format of seminar classes with small groups working together was favorable, as some students were keener (or more skilled) to use computers and mathematical software. The lecture conducted in the 3rd class was followed with close attention, as students knew that in the next seminar class, the methods presented would be needed. The contents of the lecture are those included in Supporting Information Material. The underlying theory of continuous sedimentation of concentrated broths was introduced. Socratic questions were used to foster participation and to promote students' reflection. The most commonly used methods for thickeners design were explained and examples were solved on the blackboard. In the last seminar session, as previously described, students were asked to solve the problem presented in this paper and report how long it took them using different methods. They used four different methods: (a) Settling flux, (b) Talmage–Fitch, (c) Merta–Ziolo, and (d) Yoshioka methods. The contact hours devoted to this unit including lectures and paper-and-pencil calculations in previous years were the same as in this approach. For the seminar class, before working with the software, a consensus was reached in class on the algorithm steps. To this end, several open-ended questions were asked (see examples in Table 3). When it came to the use of SMath, once students understood how to code the necessary actions, it took them, on average, 1.5 hr to solve the problem using the total flux method. Subsequently, the 2-hr seminar (last session) was not enough to solve the

TABLE 4 Student questionnaire on the use of SMATH to calculate the area of a thickener

Q1	I recommend the use of SMATH to solve engineering problems instead of the traditional approaches
Q2	With the use of SMATH, I can better understand the theory about engineering problems
Q3	I think that, from now on, SMATH will be very useful in my academic and professional career
Q4	SMATH is a simple and easy programme to use
Q5	The knowledge acquired in this course about SMATH is enough to solve other engineering problems raised in other subjects
Q6	I want to continue deepening my knowledge of this software
Q7	I would like to learn about other mathematical software
Q8	In general, I am very satisfied with the use of SMATH in this course

Note: Possible answers are as follows: 1, totally disagree; 2, disagree; 3, agree; 4, totally agree.

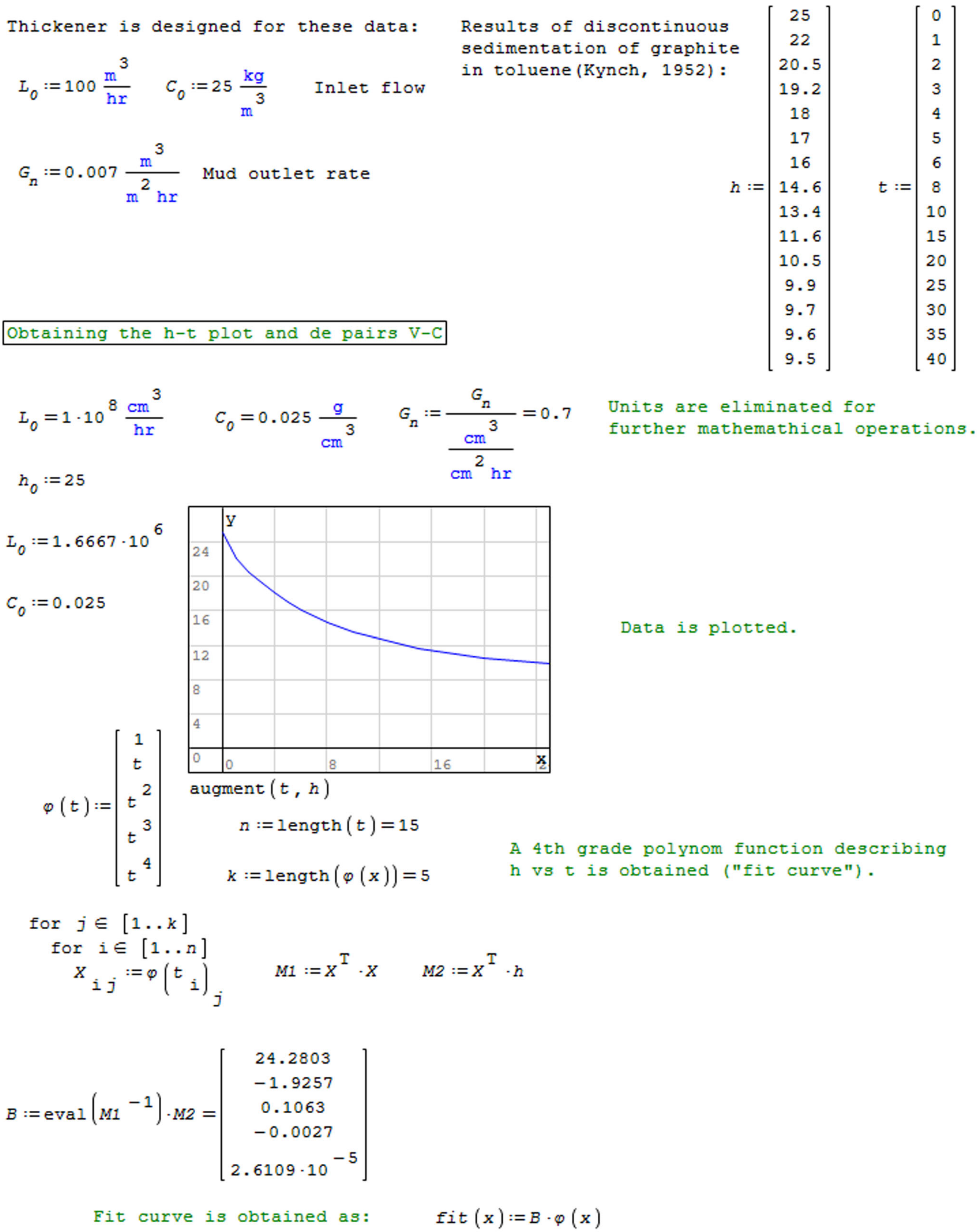


FIGURE 2 The thickener design in SMath part 1 of 4. Fitting experimental *h-t* values to a 4th-degree polynomial

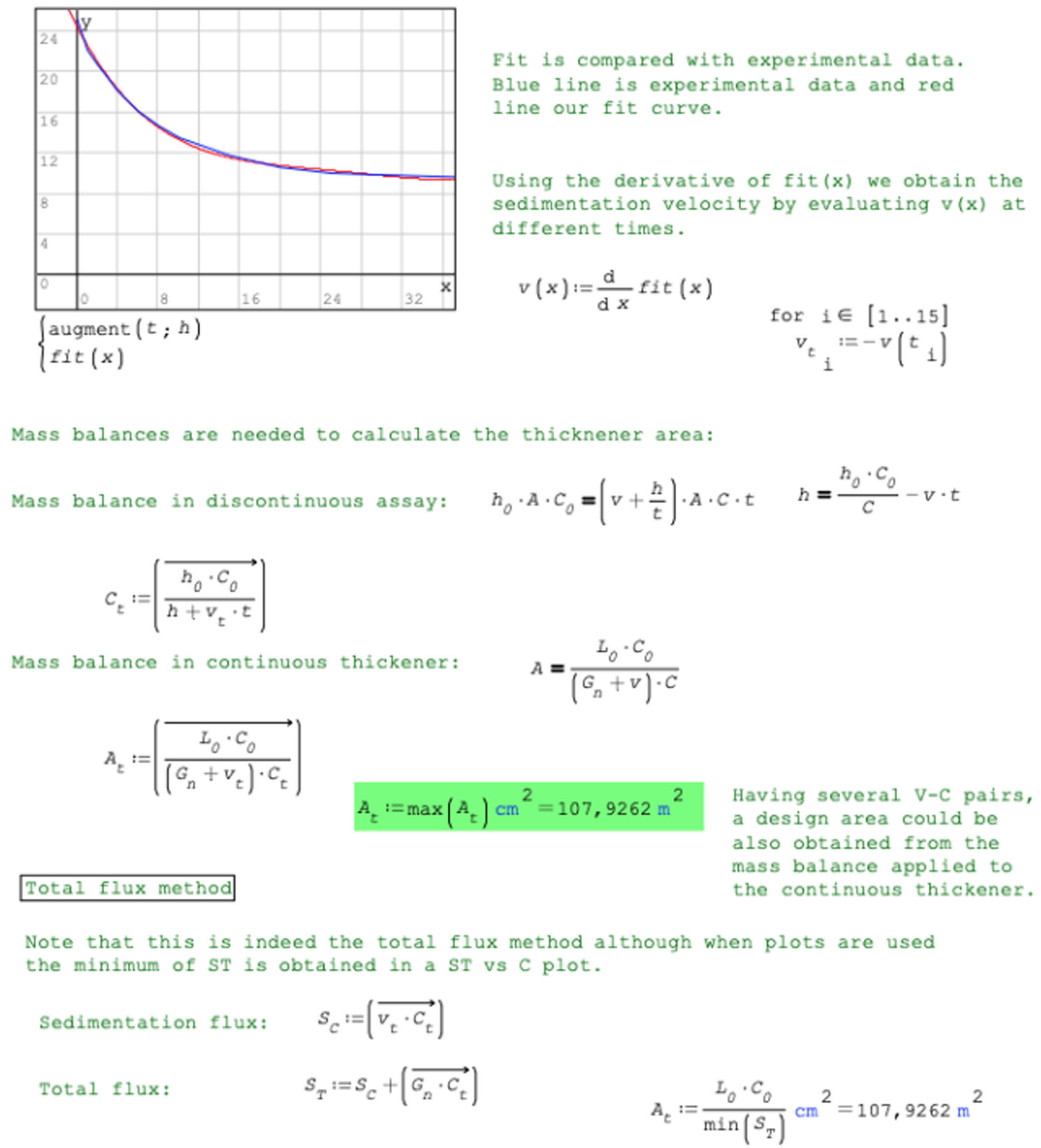


FIGURE 3 The thickener design in SMath part 2 of 4. Settling velocity calculation and mass balances (total flux method)

problem with the four proposed methods, which were therefore applied autonomously in groups. Students self-reported times between 5 and 8 hr to complete the task. During this period, several tutoring sessions on how to use the software and the theoretical basis of thickeners were needed.

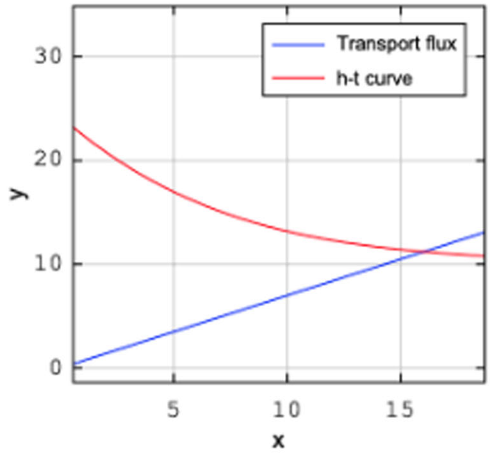
In Figures 2–5, the expected outcome is shown. As can be seen, the straightforward syntax of SMath can be easily followed without any previous knowledge. The resolution steps are commented in green. In Figure 2, a 4th-degree polynomial is obtained from the bibliographic data of graphite in toluene. The interphase height (h) and

Merta and Ziolo method:

Solution is obtained using
the time when transport flux
and h-t curve encounter.

Mathematically,
it can be obtained as:

$t_n := \text{solve}(fit(x) - G_n \cdot x ; x) = 16{,}0238$



$\begin{cases} G_n \cdot x \\ fit(x) \end{cases}$

Then the solution is: $A_t := \frac{L_0}{h_0} \cdot t_n \ \text{cm}^2 = 106{,}8275 \ \text{m}^2$

Talmage-Fitch method

An Area must be supposed and iteration must continue until the deviation with the calculated A be acceptable.

$A := 90 \cdot 10^4$ supposed area $\qquad G_n = \frac{L_n}{A} \qquad L_n := G_n \cdot A = 6{,}3 \cdot 10^5$

$C_n := \frac{L_0 \cdot C_0}{L_n} = 0{,}0661 \qquad h_u := \frac{C_0 \cdot h_0}{C_n} = 9{,}4498$

$t_u := \min(\text{solve}(fit(x) - h_u ; x ; 0; 1000)) = 32{,}5813$

$A_t := \frac{t_u}{h_0 \cdot C_0} \cdot L_0 \cdot C_0 = 2{,}1721 \cdot 10^6$ calculated area

$A := 100 \cdot 10^4$

$$A_t := \frac{\min\left(\text{solve}\left(fit(x) - \frac{C_0 \cdot h_0}{\frac{L_0 \cdot C_0}{G_n \cdot A}} ; x ; 0; 1000\right)\right)}{h_0 \cdot C_0} \cdot L_0 \cdot C_0 \ \text{cm}^2 = 142{,}3202 \ \text{m}^2$$

$A := 105 \cdot 10^4$

$$A_t := \frac{\min\left(\text{solve}\left(fit(x) - \frac{C_0 \cdot h_0}{\frac{L_0 \cdot C_0}{G_n \cdot A}} ; x ; 0; 1000\right)\right)}{h_0 \cdot C_0} \cdot L_0 \cdot C_0 \ \text{cm}^2 = 114{,}1409 \ \text{m}^2$$

$A := 106{,}8 \cdot 10^4$

$$A_t := \frac{\min\left(\text{solve}\left(fit(x) - \frac{C_0 \cdot h_0}{\frac{L_0 \cdot C_0}{G_n \cdot A}} ; x ; 0; 1000\right)\right)}{h_0 \cdot C_0} \cdot L_0 \cdot C_0 \ \text{cm}^2 = 106{,}9284 \ \text{m}^2$$

FIGURE 4 The thickener design in SMath part 3 of 4. Thickener design area calculation using Merta and Ziolo and Talmage–Fitch methods

Yoshioka method

$$C_{max} := \frac{C_0 \cdot h_0}{\min(h)} = 0{,}0658$$

From the maximum concentration, a tangent to Sc-C curve should indicate the critical sedimentation flux.

$$\varphi\left(C_t\right) := \begin{bmatrix} 1 \\ C_t \\ {C_t}^2 \\ {C_t}^3 \\ {C_t}^4 \\ {C_t}^5 \\ {C_t}^6 \end{bmatrix}$$

Here we need a fit for: Sc-C.
A 6th grade polynom was used.

$$n := \text{length}(t) = 15$$

$$k := \text{length}(\varphi(x)) = 7$$

```
for j ∈ [1..k]
  for i ∈ [1..n]
    X_{i j} := φ(C_t_i)_j
```

$$M1 := X^T \cdot X \qquad M2 := X^T \cdot S_C$$

$$B2 := \text{eval}\left(M1^{-1}\right) \cdot M2$$

$$fit2(x) := B2 \cdot \varphi(x)$$

$$m(x) := \frac{d}{dx} fit2(x)$$

```
for i ∈ [1..12]
  m_i := m(C_t_i)
```

Thus we obtain tangent lines for different solid concentration values.

The following program allows obtaing the tangent line which starts at the Cmax and calculates the critical sedimentation flux:

Critical Gs calculator
Inputs:Sc-Ct fit, Ct,Cmax,tangent lines, error tolerance.

```
CALC_G_sCritic(f(x); a; b; c; tolerance) := for i ∈ [1..12]
                                               o := -f(a_i)/(b - a_i) - c_i
                                               if |o| < tolerance
                                                 G_scritic := f(a_i) - c_i · a_i
                                                 break
                                               else
                                                 continue
                                             G_scritic
```

Then, design area is calculated as:

$$A_t := \frac{L_0 \cdot C_0}{CALC\_G_{sCritic}\left(fit2(x);\, C_t;\, C_{max};\, m;\, 0{,}3\right)} \text{ cm}^2 = 113{,}85 \text{ m}^2$$

FIGURE 5 The thickener design in SMath part 4 of 4. Thickener design area calculation using Yoshioka method time (t) were introduced as one-column matrixes and then plotted using the "augment" function, creating a two-column matrix. In Figure 3, the fit curve was compared with the experimental data. The fit curve derivative (obtained with a SMath-integrated syntax) was then evaluated at different time data. To do so, matrix positions can be called, adding a numerical suffix. In the example, a numerical range (1–15) was created and then used to call the different matrix positions using the "for" code block. This provided the sedimentation velocity (V_t) for each time data. Moreover, in Figure 3, mass balances were used to calculate the continuous thickener area from the pair V_t–C_t (concentration at t time). The total flux method was then applied using the V_t and C_t matrixes. The latter was obtained from the mass balances performed with the discontinuous assay. In Figure 4, the Merta and Ziolo method was followed. Thus, an integrated nonlinear equation solver was used to find the intersection of the Fit curve with the Transport flux equation. Furthermore, in Figure 4, the required steps to apply the Talmage–Fitch method are displayed. As this method required the mud outlet concentration to calculate the underflow height (h_u) and then the critical time (t_u), an iterative solution algorithm was used (only three iterations were needed). The Equation solver was nested with a minimum ("min") function, as the solver could give more than one solution. In Figure 5, the Yoshioka method was applied. Here, a 6th-degree polynomial was used. The method is based on obtaining a critical settling flux by drawing a tangent line to the settling flux curve from the maximum attainable mud concentration. The intersection would indicate the critical settling flux, from which the design area is calculated. In SMath, a simple custom-made function was produced to obtain such a critical value. The function determines the tangent line (contained in the m matrix) that intersects the x axis (mud concentrations), and then the critical settling flux is calculated. From the SMath implementation (Figures 2–5), it seems quite obvious that the Total Flux or the Merta-Ziolo methods allow for a shorter resolution time. The Talmage–Fitch and Yoshioka methods are more easily applied when the underflow concentration is imposed. If this was the case, resolution difficulty would be similar. Intentionally, in the proposed problem, the mud extraction velocity was a design requirement instead. This forces the assumption of an area value and iteration (e.g., in the Talmage–Fitch method; Figure 4) or application of programming (e.g., in the Talmage–Fitch method; Figure 5). As can be seen from the results, all methods, except Yoshioka's, were equally accurate. Yoshioka's method required the estimation of the maximum attainable mud concentration. Besides, critical flux was calculated from a tangent line that was obtained

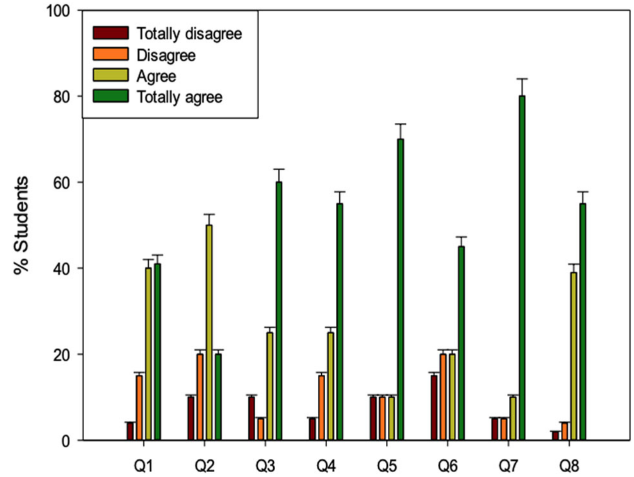


FIGURE 6 Students' answers to the survey

numerically. However, the design area calculated was only 6% greater. In view of the steps in the SMath Studio document, several mathematical and programming concepts were also taught and applied during the activity.

With regard to the postcourse survey (answers are shown in Figure 6), it is necessary to emphasize the degree of satisfaction of most students with the use of SMath in class. The use of SMath allowed students to understand the theoretical basis of thickener design, increasing their motivation to use this software to solve other engineering problems that may be presented in this and other subjects. SMath language is more natural than other software solutions such as Matlab, and the apparent simplicity was perceived by students as less intimidating. As it can be observed in Figure 6, the majority of students (65% and 90%) would be interested in deepening the learning of this and similar software (Q6 and Q7, respectively). Q8 of the survey gave an approximate percentage of the degree of satisfaction after having used SMath to solve engineering problems: 55% of them were very satisfied, whereas 39% were satisfied.

4 | CONCLUSIONS

SMath Studio was probed as a lightweight alternative to MathCad and Matlab for academic purposes. This software was used satisfactorily to teach thickener's design to students without previous knowledge of this or similar mathematical software. The proposed seminar problem was solved with SMath Studio using different design methods. Thus, several mathematical and programming concepts were also taught. Unlike the pencil-and-paper resolution exercises, this approach fostered critical thinking, teamwork, and the development of other skills (computers and mathematical software). As no programming language was needed, only two 2-hr introductory classes were enough to solve problems in class. Student's satisfaction with the methodology was high and complementary competences related with computers and programming could be also covered within the course.

ACKNOWLEDGMENTS

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NOMENCLATURE

- A thickener critical area in continuous operation (m^2)
- C solids concentration between the feed (C_0) and the underflow (C_u) (g/L)
- C_0 initial concentration of solids (g/L)
- C_u underflow concentration of solids (g/L)
- CE chemical engineering
- CEE chemical engineering education
- G_n outlet mud velocity (m/s)
- h height in the batch test (m)
- L_0 liquid feed flow (m^3/s)
- L_n mud outlet flow (m^3/s)
- PLE project-led engineering
- S_C solids flux at a C concentration (number of particles per volume of dispersion) ($g/m^2 \cdot s$)
- S_T total downward flux of solids including settling and transport ($g/m^2 \cdot s$)
- t time in batch operations (s)
- t_n Merta–Ziolo's critical time (required for settling of all solids) (m/s)
- t_u Talmage–Fitch's critical time (required for settling of all solids) (m/s)
- V_C settling velocity at a given concentration (m/s)
- V_t settling velocity at a given concentration (obtained for a time t) (m/s)

ORCID

Juan J. Gallardo-Rodríguez http://orcid.org/0000-0003-2953-7699

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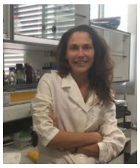
AUTHOR BIOGRAPHIES

Juan J. Gallardo-Rodríguez currently, Assistant Professor at Department of Chemical Engineering of the University of Almería. His research trajectory has been focused on microalgal bioprocessing for high value bioactives production. Additionally, he has some experience in other biotechnological fields such as water treatment, bioremediation and animal cell culture. The main area, however, has been the design, characterization, optimization and scaling of culture systems for photosynthetic microorganisms, where he has accumulated 27 JCR works and more than 400 citations. He has participated in several projects of competitive public and private financing since 2007. From 2017 to 2019, at his former affiliation (Assistant Professor at the University of Concepción, Chile), he was IP of a project aimed at making dinoflagellates microalgae cell factories for drug production.

Ana Ruiz-Ortega is social psychologist with a Master Degree in Human Resources Management and Development. Ph.D. student at the University of Jaén (Spain; social psychology doctorate degree). Currently working as Assistant Professor at the Faculty of Education and Sciences of the University Andrés Bello (Concepción, Chile). Her research interest involve Emotional Intelligence, Frustration and their impact on learning process. She have coauthored research papers on learning process evaluation. Additionally, she have a wide work experience in quality assurance systems.



Elvira Navarro-López, Chemical Engineer by University of Almería (2009). In addition, she is also Technical Industrial Engineer by the University of Jaén (2009). In 2011, she awarded a FPI grant from "Ministerio de Ciencia y Tecnología" to carry out doctoral thesis in the University of Almería focused on production of biodiesel by enzymatic transesterification reactions. During this period, she combined the work at the laboratories of the chemical engineering department with the obtention of several postgraduate degrees, such as, Master of Solar Energy (2010), Master in Biotechnology and Industrial Bioprocess (2011), Master in occupational Risk Prevention CV date 13/01/2020 2 (2013), Master in Chemical Engineering (2015) and Doctoral dissertation (2016, cum laude distinction) all of them in the university of Almería. The Ph.D. Thesis was awarded by international PhD School, University of Almería in May (2016). During the Ph.D. grant, she has been working in the production of biodiesel from lipids obtained from microalgae biomass by enzymatic transesterification reactions, resulting in the publication of four papers that make up her doctoral thesis. Moreover, she has participated in teaching task (350 hr from 2011–2012 to the current academic year), teaching innovation projects, articles, evaluation committees and several scientific dissemination activities for high school students. From April 2017, she has been working as post-doctoral researcher in the European project SABANA "Sustainable Algae Biorefinery for Agriculture aNd Aquaculture" collaborating with other research groups in the obtention of several bioproducts from microalgae cultivated in wastewater for agriculture and aquaculture applications, such as the Karlsruhe Institute of Technology (KIT) and the National Laboratory of Energy and Geology (LNEG) from Lisbon. These activities have been reflected in 15 scientific papers published (three under review), one accepted book chapter, 25 contributions both at national and international congresses and she has actively worked as a researcher on one national projects and one European ProjecT and as collaborator in almost three national projects more.

Maria del Carmen Cerón-García, María del Carmen Cerón García, Bachelor of Science in Chemical Sciences, specialized in Industrial Chemistry and PhD in Chemical Engineering. Associate Professor in the Department of Chemical Engineering at the University of Almeria has been developing its professional activity in this university, both in the field of teaching and research, for two decades. The main line of his research is in various aspects of the fields of bioprocess engineering, modeling and operation, and subsequent processing. His work has been mainly related to the application of bioprocess engineering for the cultivation of marine microalgae. Above all, the cultivation of microalgae in the three modes of nutrition, photo-, mixo-, and heterotrophs for different purposes and physical-chemical methods for the recovery of high added value products both in indoor and outdoor. She has been researching for the identification, extraction and production of microalgae carotenoids for more than 15 years. In the last 5 years, it has been mainly focused on the valorization of microalgal biomass, especially to obtain high added value compounds, through the development of processes to obtain fractions of microalga oil (polyunsaturated fatty acids) and carotenoids or even bioactives compounds from dinoflagellates, all of them with different application in cosmetics, pharmacy, biomedicine or aquaculture. She currently participates in the processing tasks in the European Project "Sustainable algae biorefinery for agriculture and aquaculture (SABANA)" and in a national project about Biotechnology of marine dinoflagellates. Production and valorization of its biomass at pilot scale. Additionally, she has been director of two thesis related to aquaculture (culture optimization, outdoor biomass production and evaluation of microalgal biomass in juveniles) and is currently director of a doctoral thesis that is being developed in the Department of Chemical Engineering related with the valorization of microalgal biomass in obtaining bioactive compounds, PUFAs and carotenoids from dinoflagellates. She has been Vice Dean of Research and Technological Innovation, degree in Chemical Engineering from the Faculty of Experimental Sciences of the University of Almeria from March 24, 2009 to December 5, 2012 and Coordinator of the Master in Chemical Engineering at the UAL of the University of Almeria since its implementation September 22, 2014 until today. The researcher has three years of research granted, approved between 1999 and 2016. In this time, he has directed seven final thesis projects in the area of chemical engineering, 17-bachelor thesis and master thesis, and two doctoral theses, being pending to defend two theses more. In this time, he has published 61 JCR articles and an international book chapter, with 2,318 citation (scopus), with an average in the last 5 years of 265 citations/year (2015–2019) (Scopus). The percentage of publications in the first quartile is higher than 75% Q1:47 and Q2:14, with the researcher's index h of 25 (Scopus), 25 (google scholar), ih-10: 36 (google scholar).

Alba Beas-Catena completed her degree in Chemical Engineering at the University of Almeria (Almeria, Spain) and at Wageningen University (Wageningen, The Netherlands), where she did her Master Thesis in the subdepartment of Environmental Technology. She got her Ph.D. in the field of Biotechnology and Industrial Bioprocesses at the University of Almeria, working with insect cells to produce a baculovirus to be used as a biopesticide. As part of her research, she has published five papers in reputed journals.

Asterio Sánchez-Mirón, Ph.D. in Chemical Engineering (2001) by the University of Almería (UAL). FPU grant from 1998 to 2001. Associated Professor in the Department of Chemical Engineering of the UAL from 2001 to 2005. Doctor Associate Professor from 2005 to 2010. Associate Professor in Chemical Engineering from 2010 to 2020, Professor from 6/2020. His research activity has focused on Biochemical Engineering and Development of Bioprocesses, with interesting scientific and technological contributions in the following areas: (a) marine microalgae for the production and purification of lipids of interest (e.g., polyunsaturated fatty acids) or biodiesel production; (b) in vitro culture of marine sponges for the production of molecules with cytotoxic activity; (c) marine dinoflagellates for the production of toxins and bioactives of commercial interest; (d) hybridomas for the production of monoclonal antibodies; (e) production of baculoviruses from insect cells, for use as bioinsecticides. The research group to which he is enrolled has obtained important breakthroughs in marine dinoflagellate-based bioprocesses through an intense work in the quantification of the sensitivity to turbulence developed in photobioreactors (PBRs), mitigation of the cell damages caused by it, CFD-based hydrodynamics studies of PBRs, custom-made photobioreactor designs for shear-sensitive species, genetic algorithm-based culture medium formulations and evaluation and mitigation of biofouling in PBRs. He has coauthored 67 research papers published in journals included in the *Journal Citation Reports* catalog of the Science Citation Index (36 in the first quartile), two book chapters in publishers of recognized international prestige and two invention patents. He has participated in more than 75 communications to national and international conferences of special relevance, with seniority and periodicity. Several of these contributions have been papers or invited papers in which he has been coauthor and in four of them at international conferences he has made the oral presentation. The current h index is 23. He has supervised three Doctoral Theses. He has participated in 18 projects of competitive public financing, four of public regional financing (two as IP), in two European funding projects, two Chilean and in three private financing projects. All of them have pharmaceutical and industrial interest.

Francisco García-Camacho, Ph.D. in Chemical Sciences (1991) by the University of Granada (UGR). PFPI grant of the Junta de Andalucía from 1988 to 1989 (Campus Universitario de Almería, UGR). Interim Prof. of E.U. and University in the Department of Chemical Engineering of the UGR until 1993 (University Campus of Almeria) and the University of Almeria (UAL) until 1995. Professor of the UAL from 1995 to 2010. In January of that same year reaches the Professorship in Chemical Engineering. His research activity has focused on the area of Biochemical Engineering and development of Bioprocesses, with interesting scientific and technological contributions in the following areas: (a) marine microalgae for the production and purification of lipids of interest (e.g., fatty acids) polyunsaturated) or biodiesel production; (b) in vitro culture of marine sponges for the production of molecules with cytotoxic activity; (c) marine dinoflagellates for the production of toxins and bioactives of commercial interest; (d) hybridomas for the production of monoclonal antibodies; (e) production of baculoviruses, from insect cells, for use as bioinsecticides. He has coauthored 121 research papers published in journals included in the *Journal Citation Reports* of the Science Citation Index, six book chapters in publishers of recognized international prestige and four invention patents. The current h index is 35. He has directed 10 Doctoral Theses and seven competitive national public funding projects. He has participated in six projects at regional and national level, three European and collaborated in seven contracts with national and foreign companies. In recent years, the research team that he leads have achieved important breakthroughs in the culture of marine dinoflagellates through an intense work in the quantification of the sensitivity to turbulence developed in photobioreactors, mitigation of the damages caused by it and in the design of specific culture medium. The studies carried out have allowed to improve the yield of the cultures and advance in the scale-up strategies. The future work of the research team will focus on the development of specific protocols and methodologies of bioprocess engineering, that will allow (a) to select in a simple and fast way those sensitive species that are cultivable, (b) determine the optimum conditions of cultivation and reactor design, (c) development of new formulations of culture media that maximize the productivity of substances of interest and (d) proof of concept on a semiindustrial scale. In relation to his research training capacity, since 1993 he has been coordinator of several doctoral programs at the University of Almeria closely related to his scientific activity: (a) "Biochemical Engineering" (from 1993/1994 to 2000/2001); (b) "Bioprocess Engineering: drugs, environment and food" (from 2001/2002 to 2003/2005); (c) "Bioprocess Engineering and Industrial Biotechnology" (from 2005/2006 to 2010). He has promoted the creation of the Doctoral Program "Biotechnology and Industrial Bioprocesses Applied to Agri-Food and the Environment" of the UAL (mention of excellence Ref. MEE2011-0197), being a member of the Academic Committee thereof from 2011 to date. He also coordinated the Official Master's Degree "Industrial and Agri-Food Biotechnology" of the UAL, from its implementation in 2010 until 2015.

SUPPORTING INFORMATION

Additional supporting information may be found online in the Supporting Information section.

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