$$y'' + y = \frac{1}{\cos(x)}$$

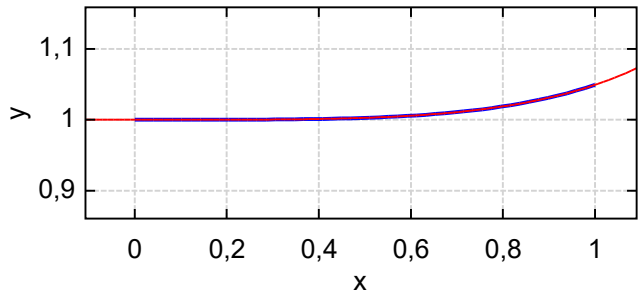
$y(x) := \cos(x) + x \cdot \sin(x) + \cos(x) \cdot \ln(\cos(x))$ $AbsTol := 10^{-7}$ $RelTol := 10^{-7}$

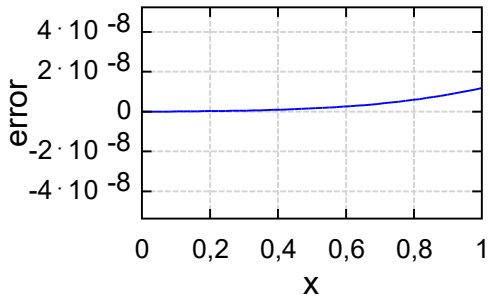
$y'(x) := \frac{d}{dx} y(x)$ $y'(x) = x \cdot \cos(x) - \sin(x) \cdot (2 + \ln(\cos(x)) - 1)$

$N := 50$ $\begin{bmatrix} x_{min} & x_{max} \end{bmatrix} := \begin{bmatrix} 0 & 1 \end{bmatrix}$ $y_0 := \text{stack}(y(0), y'(0)) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$res := \text{mk52lfa}\left(y_0, x_{min}, x_{max}, N-1, D(x, y) := \text{stack}\left(y_2, \frac{1}{\cos(x)} - y_1\right)\right)$

$y := res_{[1..N]\begin{bmatrix} 1 \\ 2 \end{bmatrix}}$ $xn := \text{col}(res, 1)$ $err := \text{augment}\left(xn, \text{col}(res, 2) - \overrightarrow{y(xn)}\right)$





$\begin{cases} y \\ y(x) \end{cases}$

$$\left(1 + x^2\right) \cdot y'' + (y')^2 + 1 = 0$$

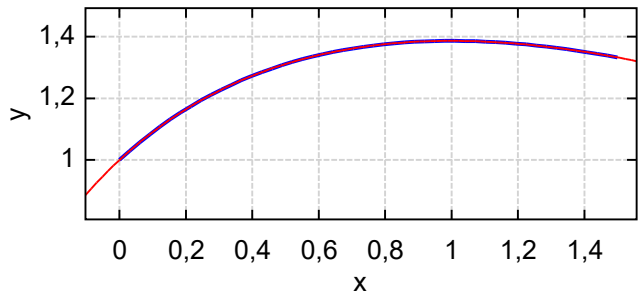
$y(x) := 1 - x + 2 \cdot \ln(1 + x)$ $AbsTol := 10^{-3}$ $RelTol := 10^{-3}$

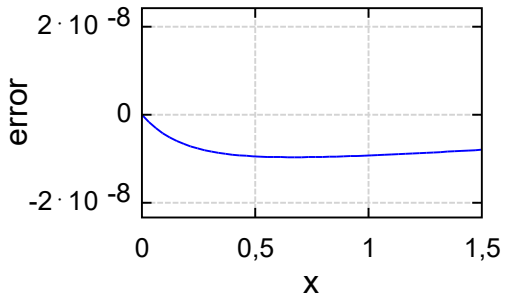
$y'(x) := \frac{d}{dx} y(x)$ $y'(x) = -\frac{-1 + x}{1 + x}$

$N := 50$ $\begin{bmatrix} x_{min} & x_{max} \end{bmatrix} := \begin{bmatrix} 0 & 1.5 \end{bmatrix}$ $y_0 := \text{stack}(y(0), y'(0)) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$res := \text{rkm9mka}\left(y_0, x_{min}, x_{max}, N-1, D(x, y) := \text{stack}\left(y_2, -\frac{1 + y_2^2}{1 + x^2}\right)\right)$

$y := res_{[1..N]\begin{bmatrix} 1 \\ 2 \end{bmatrix}}$ $xn := \text{col}(res, 1)$ $err := \text{augment}\left(xn, \text{col}(res, 2) - \overrightarrow{y(xn)}\right)$





$\begin{cases} y \\ y(x) \end{cases}$

$$y'' + 2\cdot y' + 2\cdot y = 2\cdot e^{-x}\cdot\cos(x)$$

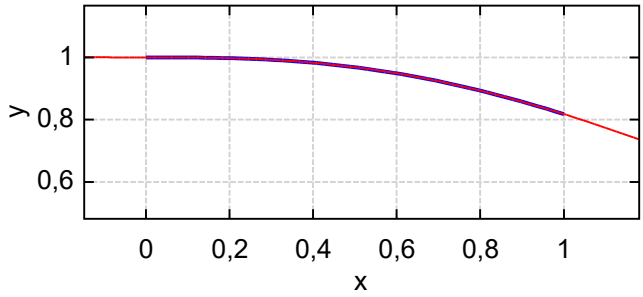
$y(x) := e^{-x}\cdot(\cos(x)+\sin(x)+x\cdot\sin(x))$ $\quad AbsTol := 10^{-6}$ $\quad RelTol := 10^{-6}$

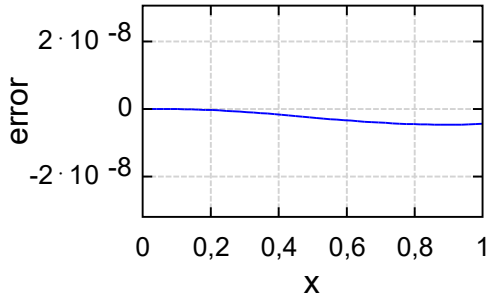
$y'(x) := \frac{d}{dx}y(x)$ $\quad y'(x) = -\frac{(1+x)\cdot(\sin(x)-\cos(x))+\cos(x)}{e^{x}}$

$N := 50$ $\quad \begin{bmatrix} x_{min} & x_{max} \end{bmatrix} := [\,0\ 1\,]$ $\quad y_0 := stack(y(0), y'(0)) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$res := mk52lfa\left(y_0, x_{min}, x_{max}, N-1, D(x,y) := stack\left(y_2, 2\cdot\left(e^{-x}\cdot\cos(x) - (y_2 + y_1)\right)\right)\right)$

$y := res_{[1..N]\begin{bmatrix} 1 \\ 2 \end{bmatrix}}$ $\quad xn := col(res, 1)$ $\quad err := augment\left(xn, col(res, 2) - \overrightarrow{y(xn)}\right)$





$\begin{cases} y \\ y(x) \end{cases}$

$$y'' + 4\cdot y = e^{3\cdot x}\cdot(13\cdot x - 7)$$

$AbsTol := 10^{-8}$ $\quad RelTol := 10^{-8}$

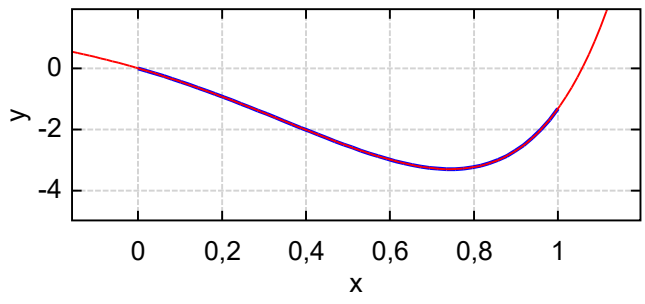
$y(x) := \cos(2\cdot x) - \sin(2\cdot x) + e^{3\cdot x}\cdot(x-1)$

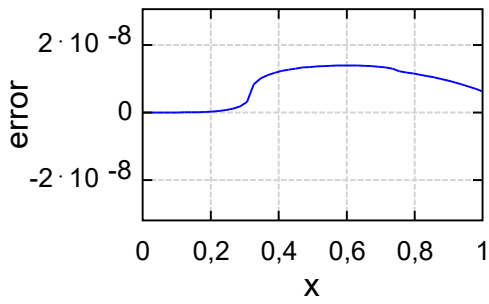
$y'(x) := \frac{d}{dx}y(x)$ $\quad y'(x) = -\left(-e^{3\cdot x}\cdot(1+3\cdot(-1+x)) + 2\cdot(\sin(2\cdot x)+\cos(2\cdot x))\right)$

$N := 50$ $\quad \begin{bmatrix} x_{min} & x_{max} \end{bmatrix} := [\,0\ 1\,]$ $\quad y_0 := stack(y(0), y'(0)) = \begin{bmatrix} 0 \\ -4 \end{bmatrix}$

$res := mk52lfa\left(y_0, x_{min}, x_{max}, N-1, D(x,y) := stack\left(y_2, e^{3\cdot x}\cdot(13\cdot x - 7) - 4\cdot y_1\right)\right)$

$y := res_{[1..N]\begin{bmatrix} 1 \\ 2 \end{bmatrix}}$ $\quad xn := col(res, 1)$ $\quad err := augment\left(xn, col(res, 2) - \overrightarrow{y(xn)}\right)$





$\begin{cases} y \\ y(x) \end{cases}$

$y'' + 4 \cdot y' + 4 \cdot y = 0$

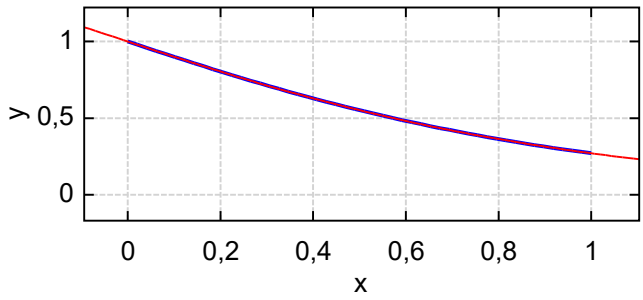
$y(x) := (1 + x) \cdot e^{-2 \cdot x}$ $\quad AbsTol := 10^{-7}$ $\quad RelTol := 10^{-7}$

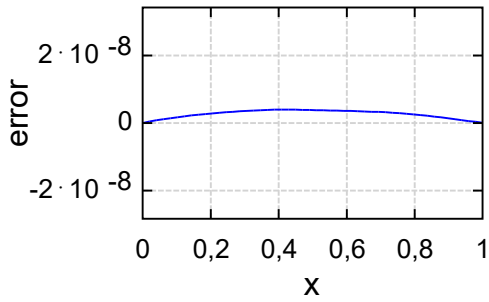
$y'(x) := \frac{d}{d\,x} y(x)$ $\quad y'(x) = \frac{1 - 2 \cdot (1 + x)}{e^{2 \cdot x}}$

$N := 50$ $\quad \begin{bmatrix} x_{min} & x_{max} \end{bmatrix} := \begin{bmatrix} 0 & 1 \end{bmatrix}$ $\quad y_0 := stack(y(0), y'(0)) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$res := mk52lfa\left(y_0, x_{min}, x_{max}, N - 1, D(x, y) := stack\left(y_2, -4 \cdot \left(y_2 + y_1\right)\right)\right)$

$y := res_{[1..N]\begin{bmatrix} 1 \\ 2 \end{bmatrix}}$ $\quad xn := col(res, 1)$ $\quad err := augment\left(xn, col(res, 2) - \overrightarrow{y(xn)}\right)$





$\begin{cases} y \\ y(x) \end{cases}$

$y'' - y = \sin(x) + \cos(2 \cdot x)$ $\quad AbsTol := 10^{-8}$ $\quad RelTol := 10^{-8}$

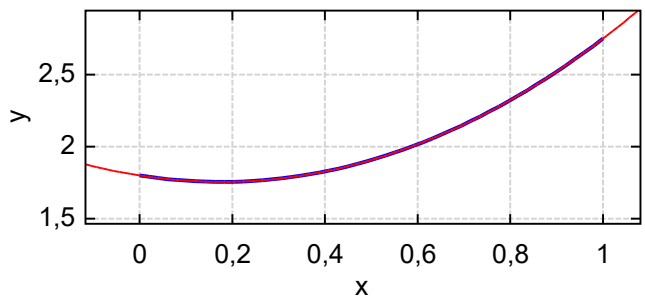
$y(x) := e^{x} + e^{-x} - 0.5 \cdot \sin(x) - 0.2 \cdot \cos(2 \cdot x)$

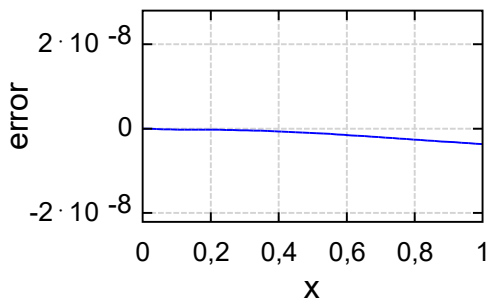
$y'(x) := \frac{d}{d\,x} y(x)$

$N := 50$ $\quad \begin{bmatrix} x_{min} & x_{max} \end{bmatrix} := \begin{bmatrix} 0 & 1 \end{bmatrix}$ $\quad y_0 := stack(y(0), y'(0)) = \begin{bmatrix} 1.8 \\ -0.5 \end{bmatrix}$

$res := mk52lfa\left(y_0, x_{min}, x_{max}, N - 1, D(x, y) := stack\left(y_2, y_1 + \sin(x) + \cos(2 \cdot x)\right)\right)$

$y := res_{[1..N]\begin{bmatrix} 1 \\ 2 \end{bmatrix}}$ $\quad xn := col(res, 1)$ $\quad err := augment\left(xn, col(res, 2) - \overrightarrow{y(xn)}\right)$





$\begin{cases} y \\ y(x) \end{cases}$

$y'' - 3 \cdot y' = e^{5 \cdot x}$

$AbsTol := 10^{-7}$ $RelTol := 10^{-7}$

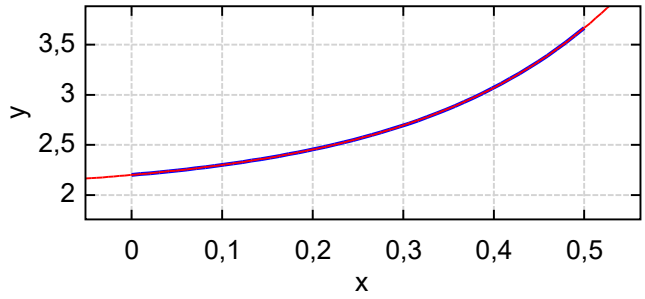
$y(x) := 2 + 0.1 \cdot \left(e^{3 \cdot x} + e^{5 \cdot x} \right)$

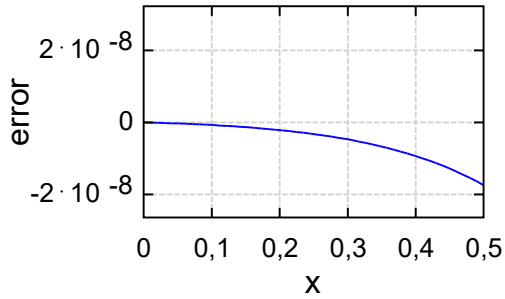
$y'(x) := \frac{d}{dx} y(x)$ $y'(x) = \frac{e^{3 \cdot x} \cdot \left(2 \cdot e^{2 \cdot x} + 3 \cdot \left(1 + e^{2 \cdot x} \right) \right)}{10}$

$N := 50$ $\begin{bmatrix} x_{min} & x_{max} \end{bmatrix} := \begin{bmatrix} 0 & 0.5 \end{bmatrix}$ $y_0 := stack(y(0), y'(0)) = \begin{bmatrix} 2.2 \\ 0.8 \end{bmatrix}$

$res := rkm9mka\left(y_0, x_{min}, x_{max}, N - 1, D(x, y) := stack\left(y_2, 3 \cdot y_2 + e^{5 \cdot x} \right) \right)$

$y := res_{[1..N]\begin{bmatrix} 1 \\ 2 \end{bmatrix}}$ $xn := col(res, 1)$ $err := augment\left(xn, col(res, 2) - \overrightarrow{y(xn)} \right)$





$\begin{cases} y \\ y(x) \end{cases}$

$y'' + 4 \cdot y = \cos(3 \cdot x)$

$AbsTol := 10^{-7}$ $RelTol := 10^{-7}$

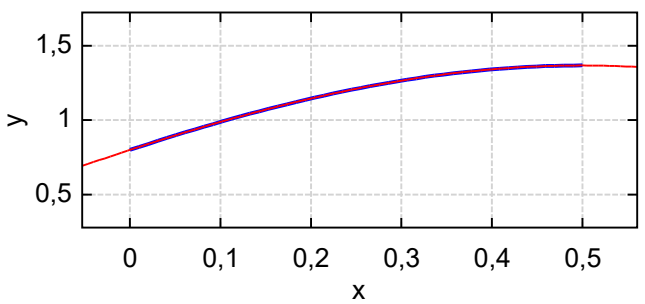
$y(x) := \cos(2 \cdot x) + \sin(2 \cdot x) - 0.2 \cdot \cos(3 \cdot x)$

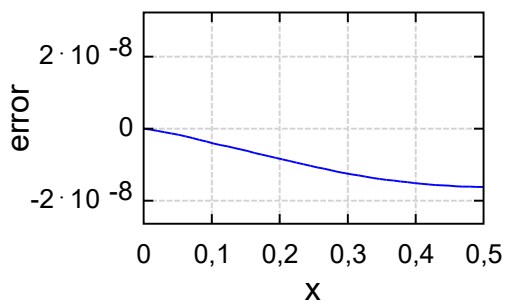
$y'(x) := \frac{d}{dx} y(x)$ $y'(x) = \frac{3 \cdot \sin(3 \cdot x) + 10 \cdot \left(-\sin(2 \cdot x) + \cos(2 \cdot x) \right)}{5}$

$N := 50$ $\begin{bmatrix} x_{min} & x_{max} \end{bmatrix} := \begin{bmatrix} 0 & 0.5 \end{bmatrix}$ $y_0 := stack(y(0), y'(0)) = \begin{bmatrix} 0.8 \\ 2 \end{bmatrix}$

$res := mk52lfa\left(y_0, x_{min}, x_{max}, N - 1, D(x, y) := stack\left(y_2, \cos(3 \cdot x) - 4 \cdot y_1 \right) \right)$

$y := res_{[1..N]\begin{bmatrix} 1 \\ 2 \end{bmatrix}}$ $xn := col(res, 1)$ $err := augment\left(xn, col(res, 2) - \overrightarrow{y(xn)} \right)$





$\begin{cases} y \\ y(x) \end{cases}$

$y'' - y' - 6 \cdot y = 2 \cdot e^{4 \cdot x}$

$AbsTol := 10^{-8}$ $RelTol := 10^{-8}$

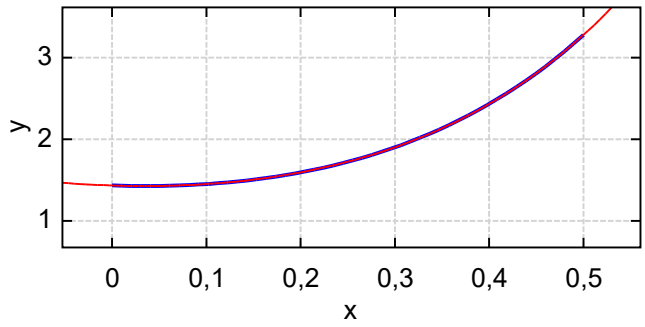
$y(x) := 0.1 \cdot e^{3 \cdot x} + e^{-2 \cdot x} + \frac{1}{3} \cdot e^{4 \cdot x}$

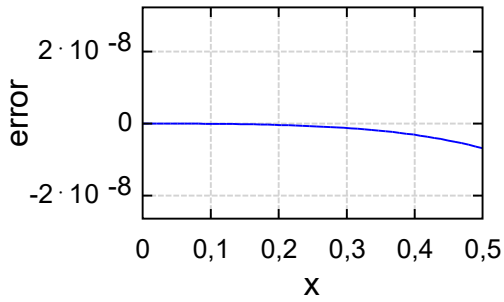
$y'(x) := \frac{d}{dx} y(x)$ $y'(x) = \frac{15 \cdot e^{5 \cdot x} \cdot (1 + 4 \cdot e^{x}) - 2 \cdot (3 \cdot (10 + e^{5 \cdot x}) + 10 \cdot e^{6 \cdot x})}{30 \cdot e^{2 \cdot x}}$

$N := 50$ $[x_{min}\ x_{max}] := [0\ 0.5]$ $y_0 := stack(y(0), y'(0)) = \begin{bmatrix} 1.4333 \\ -0.3667 \end{bmatrix}$

$res := mk52lfa(y_0, x_{min}, x_{max}, N - 1, D(x, y) := stack(y_2, 2 \cdot e^{4 \cdot x} + y_2 + 6 \cdot y_1))$

$y := res_{[1..N][1\ 2]}$ $xn := col(res, 1)$ $err := augment(xn, col(res, 2) - \overrightarrow{y(xn)})$





$\begin{cases} y \\ y(x) \end{cases}$

$y'' - 2 \cdot y' + y = 5 \cdot x \cdot e^{x}$

$AbsTol := 10^{-8}$ $RelTol := 10^{-8}$

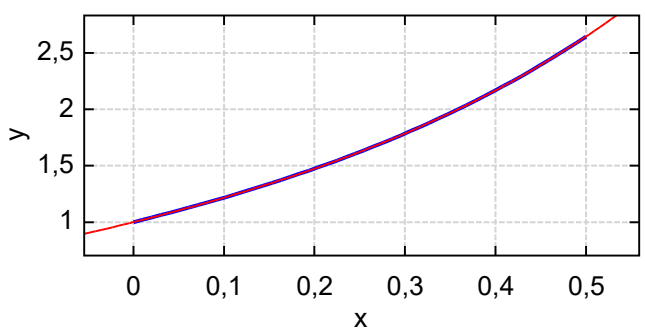
$y(x) := e^{x} + x \cdot e^{x} + 5 \cdot e^{x} \cdot \frac{x^{3}}{6}$

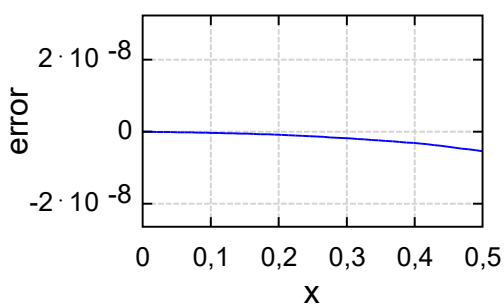
$y'(x) := \frac{d}{dx} y(x)$ $y'(x) = \frac{e^{x} \cdot (3 \cdot (2 + 5 \cdot x^{2} + 2 \cdot (1 + x)) + 5 \cdot x^{3})}{6}$

$N := 50$ $[x_{min}\ x_{max}] := [0\ 0.5]$ $y_0 := stack(y(0), y'(0)) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$res := mk52lfa(y_0, x_{min}, x_{max}, N - 1, D(x, y) := stack(y_2, 5 \cdot x \cdot e^{x} + 2 \cdot y_2 - y_1))$

$y := res_{[1..N][1\ 2]}$ $xn := col(res, 1)$ $err := augment(xn, col(res, 2) - \overrightarrow{y(xn)})$





$\begin{cases} y \\ y(x) \end{cases}$

$y'' + y' - 6 \cdot y = 3 \cdot x^2 - x - 1$

$AbsTol := 10^{-7}$ $RelTol := 10^{-7}$

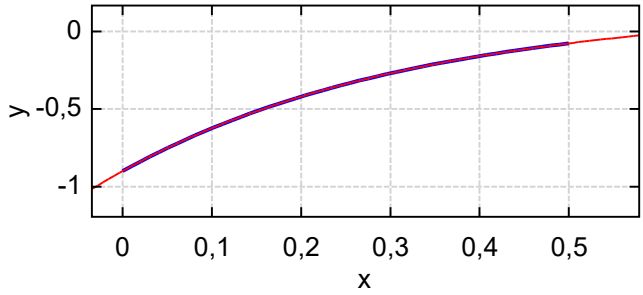
$y(x) := 0.1 \cdot e^{2 \cdot x} - e^{-3 \cdot x} - 0.5 \cdot x^2$

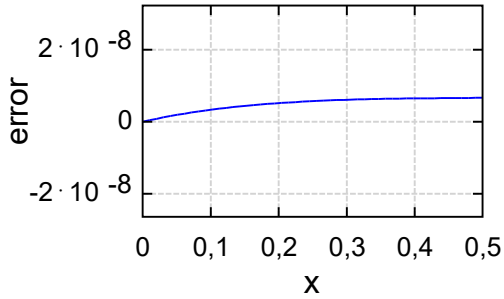
$y'(x) := \frac{d}{dx} y(x)$ $y'(x) = -\frac{5 \cdot \left(-e^{2 \cdot x} + x \cdot (2 + 3 \cdot x)\right) \cdot e^{3 \cdot x} - 3 \cdot \left(5 \cdot \left(2 + x^2 \cdot e^{3 \cdot x}\right) - e^{5 \cdot x}\right)}{10 \cdot e^{3 \cdot x}}$

$N := 50$ $\begin{bmatrix} x_{min} & x_{max} \end{bmatrix} := [0 \ 0.5]$ $y_0 := \text{stack}(y(0), y'(0)) = \begin{bmatrix} -0.9 \\ 3.2 \end{bmatrix}$

$res := \text{mk52lfa}\left(y_0, x_{min}, x_{max}, N - 1, D(x, y) := \text{stack}\left(y_2, 3 \cdot x^2 - x - 1 + 6 \cdot y_1 - y_2\right)\right)$

$y := res_{[1..N]\begin{bmatrix} 1 \\ 2 \end{bmatrix}}$ $xn := \text{col}(res, 1)$ $err := \text{augment}\left(xn, \text{col}(res, 2) - \overrightarrow{y(xn)}\right)$





$\begin{cases} y \\ y(x) \end{cases}$

$8 \cdot y'' + 2 \cdot y' - 3 \cdot y = x + 5$

$AbsTol := 10^{-6}$ $RelTol := 10^{-6}$

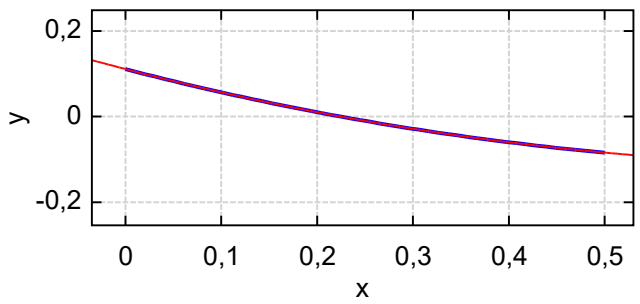
$y(x) := e^{\frac{x}{2}} + e^{-\frac{3 \cdot x}{4}} - \frac{x}{3} - \frac{17}{9}$

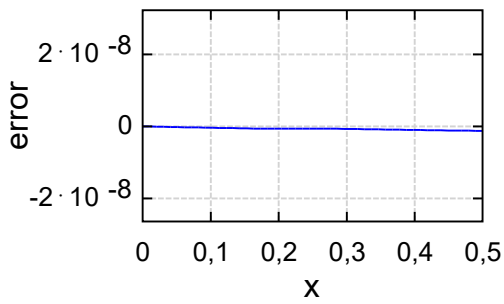
$y'(x) := \frac{d}{dx} y(x)$ $y'(x) = -\frac{3 \cdot \left(3 \cdot \left(1 + e^{\frac{5 \cdot x}{4}}\right) - x \cdot e^{\frac{3 \cdot x}{4}} + \left(7 + x - 5 \cdot e^{\frac{x}{2}}\right) \cdot e^{\frac{3 \cdot x}{4}}\right) - 17 \cdot e^{\frac{3 \cdot x}{4}}}{12 \cdot e^{\frac{3 \cdot x}{4}}}$

$N := 50$ $\begin{bmatrix} x_{min} & x_{max} \end{bmatrix} := [0 \ 0.5]$ $y_0 := \text{stack}(y(0), y'(0)) = \begin{bmatrix} 0.1111 \\ -0.5833 \end{bmatrix}$

$res := \text{mk52lfa}\left(y_0, x_{min}, x_{max}, N - 1, D(x, y) := \text{stack}\left(y_2, \frac{1}{8} \cdot \left(3 \cdot y_1 - 2 \cdot y_2 + x + 5\right)\right)\right)$

$y := res_{[1..N]\begin{bmatrix} 1 \\ 2 \end{bmatrix}}$ $xn := \text{col}(res, 1)$ $err := \text{augment}\left(xn, \text{col}(res, 2) - \overrightarrow{y(xn)}\right)$





$\begin{cases} y \\ y(x) \end{cases}$

$x^2 \cdot y'' - 2 \cdot y = 0$

$AbsTol := 10^{-8}$ $RelTol := 10^{-8}$

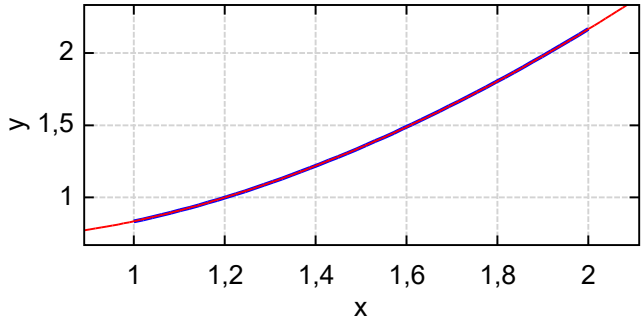
$y(x) := \frac{1}{2} \cdot x^2 + \frac{1}{3 \cdot x}$

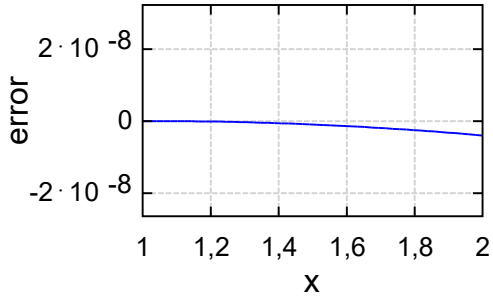
$y'(x) := \frac{d}{dx} y(x)$ $y'(x) = -\frac{1 - 3 \cdot x^3}{3 \cdot x^2}$

$N := 50$ $\begin{bmatrix} x_{min} & x_{max} \end{bmatrix} := \begin{bmatrix} 1 & 2 \end{bmatrix}$ $y_0 := \text{stack}(y(1), y'(1)) = \begin{bmatrix} 0.8333 \\ 0.6667 \end{bmatrix}$

$res := \text{mk52lfa}\left(y_0, x_{min}, x_{max}, N - 1, D(x, y) := \text{stack}\left(y_2, \frac{2 \cdot y_1}{x^2}\right)\right)$

$y := res_{[1..N]\begin{bmatrix} 1 \\ 2 \end{bmatrix}}$ $xn := \text{col}(res, 1)$ $err := \text{augment}\left(xn, \text{col}(res, 2) - \overrightarrow{y(xn)}\right)$





$\begin{cases} y \\ y(x) \end{cases}$

$y'' - 4 \cdot y' + 5 \cdot y = 3 \cdot x$

$AbsTol := 10^{-8}$ $RelTol := 10^{-8}$

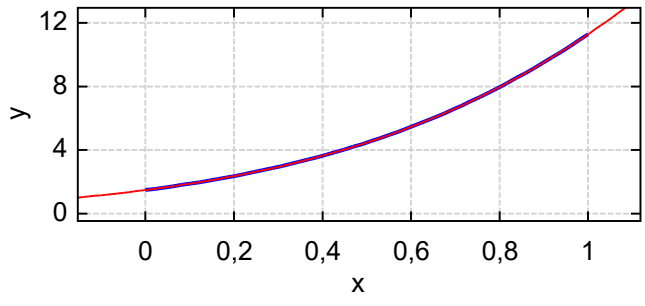
$y(x) := e^{2 \cdot x} \cdot (\cos(x) + \sin(x)) + \frac{3}{5} \cdot x + \frac{12}{25}$

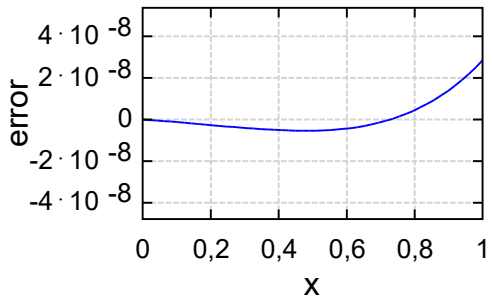
$y'(x) := \frac{d}{dx} y(x)$ $y'(x) = \frac{3 + 5 \cdot e^{2 \cdot x} \cdot (2 \cdot (\cos(x) + \sin(x)) - \sin(x) + \cos(x))}{5}$

$N := 50$ $\begin{bmatrix} x_{min} & x_{max} \end{bmatrix} := \begin{bmatrix} 0 & 1 \end{bmatrix}$ $y_0 := \text{stack}(y(0), y'(0)) = \begin{bmatrix} 1.48 \\ 3.6 \end{bmatrix}$

$res := \text{mk52lfa}\left(y_0, x_{min}, x_{max}, N - 1, D(x, y) := \text{stack}\left(y_2, 4 \cdot y_2 - 5 \cdot y_1 + 3 \cdot x\right)\right)$

$y := res_{[1..N]\begin{bmatrix} 1 \\ 2 \end{bmatrix}}$ $xn := \text{col}(res, 1)$ $err := \text{augment}\left(xn, \text{col}(res, 2) - \overrightarrow{y(xn)}\right)$





$\begin{cases} y \\ y(x) \end{cases}$

$y'' - 5 \cdot y' + 6 \cdot y = e^{x}$ $\quad AbsTol := 10^{-8}$ $\quad RelTol := 10^{-8}$

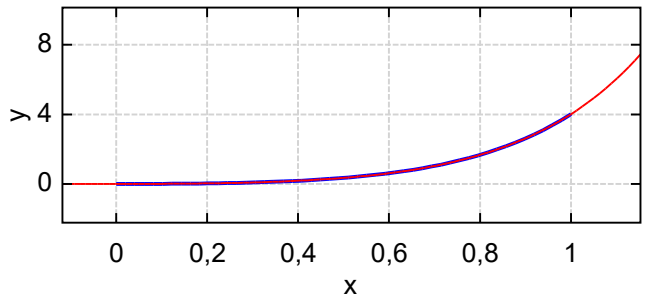
$y(x) := -e^{2 \cdot x} + 0.5 \cdot e^{3 \cdot x} + 0.5 \cdot e^{x}$

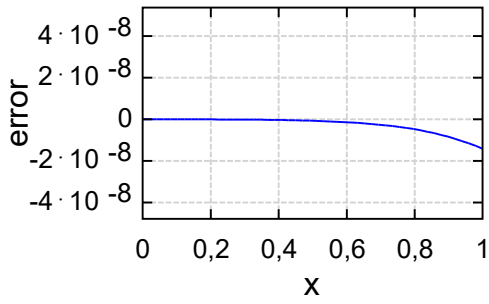
$y'(x) := \frac{d}{d x} y(x)$ $\quad y'(x) = \frac{e^{x} \cdot \left(1 + e^{x} \cdot \left(-2 + e^{x} + 2 \cdot \left(-1 + e^{x}\right)\right)\right)}{2}$

$N := 50$ $\quad \begin{bmatrix} x_{min} & x_{max} \end{bmatrix} := \begin{bmatrix} 0 & 1 \end{bmatrix}$ $\quad y_0 := stack(y(0), y'(0)) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$res := mk52lfa\left(y_0, x_{min}, x_{max}, N - 1, D(x, y) := stack\left(y_2, 5 \cdot y_2 - 6 \cdot y_1 + e^{x}\right)\right)$

$y := res_{[1..N]\begin{bmatrix} 1 \\ 2 \end{bmatrix}}$ $\quad xn := col(res, 1)$ $\quad err := augment\left(xn, col(res, 2) - \overrightarrow{y(xn)}\right)$





$\begin{cases} y \\ y(x) \end{cases}$

$y'' - 3 \cdot y' + 2 \cdot y = x^{2} + 3 \cdot x$ $\quad AbsTol := 10^{-8}$ $\quad RelTol := 10^{-8}$

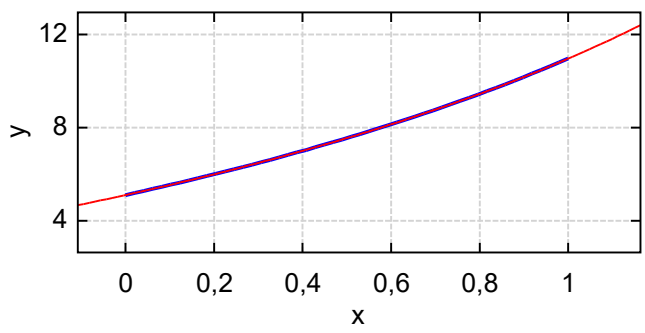
$y(x) := e^{x} + 0.1 \cdot e^{2 \cdot x} + \frac{x^{2}}{2} + 3 \cdot x + 4$

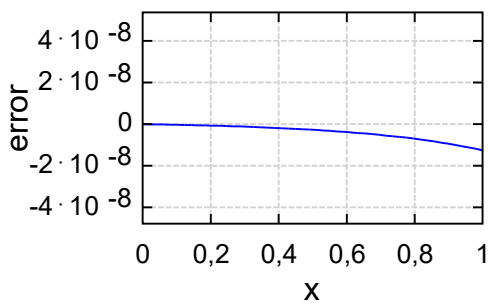
$y'(x) := \frac{d}{d x} y(x)$ $\quad y'(x) = \frac{5 \cdot \left(3 + x + e^{x}\right) + e^{2 \cdot x}}{5}$

$N := 50$ $\quad \begin{bmatrix} x_{min} & x_{max} \end{bmatrix} := \begin{bmatrix} 0 & 1 \end{bmatrix}$ $\quad y_0 := stack(y(0), y'(0)) = \begin{bmatrix} 5.1 \\ 4.2 \end{bmatrix}$

$res := mk52lfa\left(y_0, x_{min}, x_{max}, N - 1, D(x, y) := stack\left(y_2, 3 \cdot y_2 - 2 \cdot y_1 + x^{2} + 3 \cdot x\right)\right)$

$y := res_{[1..N]\begin{bmatrix} 1 \\ 2 \end{bmatrix}}$ $\quad xn := col(res, 1)$ $\quad err := augment\left(xn, col(res, 2) - \overrightarrow{y(xn)}\right)$





$\begin{cases} y \\ y(x) \end{cases}$