Transformation from Cartesian into geodetic coordinates

Plot CreateMesh γ

Convert Between Cartesian Coordinates and Geodetic Coordinates often found in geodesy and astronomy, for example, when determining the position by GPS.

The program for calculating Cartesian coordinates by geodetic coordinates has the form (λ-longitude, φ-latitude, h-geodetic height):

(WGS 84) Earth's geometrical parameters (km)

$$ax := 6378.137 \quad ay := ax \quad b := 6356.7520314245 \quad "" \quad ex := \mathrm{eval}\left(\sqrt{1-\frac{b^2}{ax^2}}\right) \quad ee := \mathrm{eval}\left(\sqrt{1-\frac{ay^2}{ax^2}}\right)$$

$$f(x) := \frac{\left(x_1\right)^2}{ax^2} + \frac{\left(x_2\right)^2}{ay^2} + \frac{\left(x_3\right)^2}{b^2} - 1$$

Geodetic(φ,λ,h) to cartesian(x,y,z) conversion

$$XYZ(\lambda, \varphi, h) := \begin{aligned} & v := \mathrm{eval}\left(\frac{ax}{\sqrt{1 - ex^2\cdot(\sin(\varphi))^2 - ee^2\cdot(\cos(\varphi))^2\cdot(\sin(\lambda))^2}}\right) \\ & x := \mathrm{eval}((v+h)\cdot\cos(\varphi)\cdot\cos(\lambda)) \\ & y := \mathrm{eval}\left(\left(v\cdot\left(1-ee^2\right)+h\right)\cdot\cos(\varphi)\cdot\sin(\lambda)\right) \\ & z := \mathrm{eval}\left(\left(v\cdot\left(1-ex^2\right)+h\right)\cdot\sin(\varphi)\right) \\ & [\,x \;\; y \;\; z\,] \end{aligned}$$

The inverse transformation is as follows:

Cartesian to geodetic coordinates conversion

$$\lambda(a) := \mathrm{arctg}\left(\frac{1}{1-ee^2}\cdot\frac{a_2}{a_1}\right)$$

$$\varphi(a) := \mathrm{arctg}\left(\frac{1-ee^2}{1-ex^2}\cdot\frac{a_3}{\sqrt{\left(1-ee^2\right)\cdot\left(a_1\right)^2+\left(a_2\right)^2}}\right)$$

where **a** is the orthogonal projection of point **A** onto the surface of the Earth's ellipsoid.

The code for calculating the orthogonal projection is borrowed from Program

"https://en.smath.com/forum/yaf_postsm69369_Draghilev-method-revisited.aspx#post69369"

Distance and projection of a point onto a surface

$$Pr(sur) := \left| \begin{array}{l} c := [1..3] \quad g_c := \dfrac{d}{d\, x_c} f(x) \quad eN := \dfrac{g}{\mathrm{norme}(g)} \quad k := [1..\mathrm{rows}(sur)] \\ d_k := \mathrm{norme}(A - \mathrm{row}(sur, k)) \quad a0 := \mathrm{eval}\left(\mathrm{row}(\mathrm{csort}(\mathrm{augment}(sur, d), 4), 1)_c^{\mathrm{T}}\right) \\ F(x) := \begin{bmatrix} f(x) \\ eN_2 \cdot (A_3 - x_3) - eN_3 \cdot (A_2 - x_2) \\ eN_3 \cdot (A_1 - x_1) - eN_1 \cdot (A_3 - x_3) \end{bmatrix} \quad a := \mathrm{eval}\left(\mathrm{al\_nleqsolve}\left(a0^{\mathrm{T}}, 10^{-11}, F(x)\right)^{(1)\,\mathrm{T}}\right) \\ \left[\, a \quad \mathrm{sign}(f(A)) \cdot \mathrm{norme}(A - a)\ \mathrm{км} \,\right] \end{array} \right.$$

Accuracy was defined as the difference between the initial (known) geodetic coordinates and the transformed coordinates

$\varepsilon h := h\# - h$ $\quad$ $\varepsilon \varphi := \varphi\# - \varphi(a)$ $\quad$ $\varepsilon \lambda := \lambda\# - \lambda(a)$

Ex.1 (GPS satellite)

$h\# := 20300\ \mathrm{км}$ $\quad$ $\lambda\# := 30\ °$ $\quad$ $\varphi\# := 55\ °$

Convert to XYZ:

$X := 13259.018058\ \mathrm{км}$ $\quad$ $y := 7655.0976448\ \mathrm{км}$ $\quad$ $z := 21830.169714\ \mathrm{км}$

Restoration of geodetic coordinates: $A := \dfrac{\mathrm{augment}(X, y, z)}{\mathrm{км}}$ $\quad$ $[\, a \quad h \,] := Pr(sur)$

$h = 2.03 \cdot 10^{4}\ \mathrm{км}$

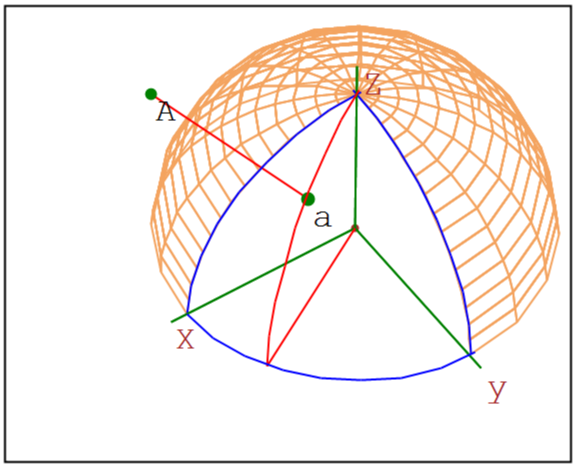
$\lambda(a) = 29.999999999\ °$

$\varphi(a) = 54.999999999\ °$

$\varepsilon h = 0.11005\ \mathrm{мм}$

$\varepsilon \varphi = 1.06565 \cdot 10^{-9}\ °$

$\varepsilon \lambda = 5.71703 \cdot 10^{-10}\ °$



Ex.2 (100000 km)

$h\# := 100000$ км $\lambda\# := 40$ ° $\varphi\# := 40$ °

Convert to XYZ:

$X := 62430.440421$ км $y := 52385.359531$ км $z := 68356.746253$ км

Restoration of geodetic coordinates: $A := \frac{\text{augment}(X, y, z)}{\text{км}}$ $[a \ h] := Pr(sur)$

$h = 1 \cdot 10^{5}$ км

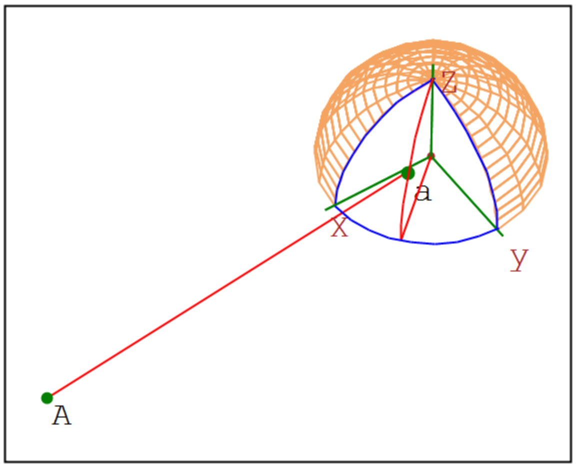
$\lambda(a) = 40.000000000$ °

$\varphi(a) = 40.000000000$ °

$\varepsilon h = -0.1618$ мм

$\varepsilon\varphi = -1.70342 \cdot 10^{-10}$ °

$\varepsilon\lambda = 2.67291 \cdot 10^{-10}$ °



Ex.3. (Superdeep Borehole)

$h\# := (-3000)$ км $\lambda\# := 40$ ° $\varphi\# := 35$ °

Convert to XYZ:

$X := 2124.2188597$ км $y := 1782.4312617$ км $z := 1917.1373296$ км

Restoration of geodetic coordinates: $A := \frac{\text{augment}(X, y, z)}{\text{км}}$ $[a \ h] := Pr(sur)$

$h = -3000$ км

$\lambda(a) = 40.000000000$ °

$\varphi(a) = 34.999999999$ °

$\varepsilon h = -0.00815$ мм

$\varepsilon\varphi = 6.85343 \cdot 10^{-10}$ °

$\varepsilon\lambda = 2.2304 \cdot 10^{-10}$ °