Eigen Vectors

$TOL := 10^{-10}$

Ex 1

$$A := \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix} \qquad [\lambda \; M] := MEigen(A) = \begin{bmatrix} \begin{bmatrix} -2 \\ -2 \\ 4 \end{bmatrix} & \begin{bmatrix} 0.4082 & 0.2673 & 0.4082 \\ -0.4082 & 0.8018 & 0.4082 \\ -0.8165 & 0.5345 & 0.8165 \end{bmatrix} \end{bmatrix}$$

$$\text{norme}(A \cdot M - M \cdot \text{diag}(\lambda)) = 1.9 \cdot 10^{-7}$$

$B := A \cdot M$ B is the matrix for change basis $\quad B \cdot \text{diag}(\lambda) \cdot B^{-1} = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$

$$\text{dn\_LinAlgEigenvalues}(A) = \begin{bmatrix} 4 \\ -2 + 1.1047 \cdot 10^{-15} \cdot i \\ -2 - 1.1047 \cdot 10^{-15} \cdot i \end{bmatrix}$$

Numerical issues in dotNumerics library

$$\text{dn\_LinAlgEigenvectors}(A) = \begin{bmatrix} -0.4082 & 0.244 - 0.407 \cdot i & 0.244 + 0.407 \cdot i \\ -0.4082 & -0.4162 - 0.407 \cdot i & -0.4162 + 0.407 \cdot i \\ -0.8165 & -0.6602 & -0.6602 \end{bmatrix}$$

Ex 2

$$A := \begin{bmatrix} 0 & 1 & -1 \\ 1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \qquad [\lambda \; M] := MEigen(A) = \begin{bmatrix} \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix} & \begin{bmatrix} 0.8165 & 0 & -0.5774 \\ -0.4082 & 0.7071 & -0.5774 \\ 0.4082 & 0.7071 & 0.5774 \end{bmatrix} \end{bmatrix}$$

$$\text{norme}(A \cdot M - M \cdot \text{diag}(\lambda)) = 2.4 \cdot 10^{-15}$$

$B := A \cdot M$

B is the matrix for change basis $\quad B \cdot \text{diag}(\lambda) \cdot B^{-1} = \begin{bmatrix} 7.8951 \cdot 10^{-16} & 1 & -1 \\ 1 & 1 & -7.2831 \cdot 10^{-16} \\ -1 & 1.5156 \cdot 10^{-15} & 1 \end{bmatrix}$

$$\text{dn\_LinAlgEigenvalues}(A) = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$$

$$\text{dn\_LinAlgEigenvectors}(A) = \begin{bmatrix} 0.8165 & -0.5774 & 1.5701 \cdot 10^{-16} \\ -0.4082 & -0.5774 & 0.7071 \\ 0.4082 & 0.5774 & 0.7071 \end{bmatrix}$$

Ex 3

$$A := \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix} \qquad [\lambda \; M] := MEigen(A) = \begin{bmatrix} \begin{bmatrix} -2 \\ -2 \\ 4 \end{bmatrix} & \begin{bmatrix} 0.4082 & 0.2673 & 0.4082 \\ -0.4082 & 0.8018 & 0.4082 \\ -0.8165 & 0.5345 & 0.8165 \end{bmatrix} \end{bmatrix}$$

$$\text{norme}(A \cdot M - M \cdot \text{diag}(\lambda)) = 1.9 \cdot 10^{-7}$$

$B := A \cdot M$ B is the matrix for change basis $\quad B \cdot \text{diag}(\lambda) \cdot B^{-1} = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$

$$\text{dn\_LinAlgEigenvalues}(A) = \begin{bmatrix} 4 \\ -2 + 1.1047 \cdot 10^{-15} \cdot i \\ -2 - 1.1047 \cdot 10^{-15} \cdot i \end{bmatrix}$$

Numerical issues in dotNumerics library

$$\text{dn\_LinAlgEigenvectors}(A) = \begin{bmatrix} -0.4082 & 0.244 - 0.407 \cdot i & 0.244 + 0.407 \cdot i \\ -0.4082 & -0.4162 - 0.407 \cdot i & -0.4162 + 0.407 \cdot i \\ -0.8165 & -0.6602 & -0.6602 \end{bmatrix}$$

Ex 4 General Numerical Issues

$$A := \begin{bmatrix} 0 & 1 & 0 & 0 \\ -u^2 & 2 \cdot u & 0 & 0 \\ -u \cdot s & s & u & 0 \\ -u \cdot t & t & 0 & u \end{bmatrix} \quad u := 1 \quad s := 2 \quad t := 3$$

This value gives an error

$u := 5$

$$w := \frac{u}{\sqrt{1 + u^2}}$$

$$[\lambda \; M] := MEigen(A) = \begin{bmatrix} \begin{bmatrix} 0.9998 + 0.0002 \cdot i \\ 0.9998 - 0.0002 \cdot i \\ 1.0002 + 0.0002 \cdot i \\ 1.0002 - 0.0002 \cdot i \end{bmatrix} & \begin{bmatrix} 0.2582 & 0.2582 + 4.0545 \cdot 10^{-5} \cdot i & 0 & 0 \\ 0.2582 + 4.0531 \cdot 10^{-5} \cdot i & 0.2582 & 0 & 0 \\ 0.5164 + 8.1063 \cdot 10^{-5} \cdot i & 0.5164 + 8.109 \cdot 10^{-5} \cdot i & 1 & 0 \\ 0.7746 + 0.0001 \cdot i & 0.7746 + 0.0001 \cdot i & 0 & 1 \end{bmatrix} \end{bmatrix}$$

$$\text{norme}(A \cdot M - M \cdot \text{diag}(\lambda)) = 0$$

Exact solution have only 3 indep eigenvectors

$$\lambda := \begin{bmatrix} u \\ u \\ u \\ u \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \qquad M := \begin{bmatrix} \frac{w}{u} & \frac{w}{u} & 0 & 0 \\ w & w & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.7071 & 0.7071 & 0 & 0 \\ 0.7071 & 0.7071 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\text{norme}(A \cdot M - M \cdot \text{diag}(\lambda)) = 0$$

Dot numerics values

$$\text{dn\_LinAlgEigenvalues}(A) = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \qquad \text{dn\_LinAlgEigenvectors}(A) = \begin{bmatrix} 0 & 0 & 0.2887 & -0.1449 \\ 0 & 0 & 0.2887 & -0.1449 \\ 1 & 0 & 0.4082 & -0.4947 \\ 0 & 1 & 0.8165 & -0.8445 \end{bmatrix}$$

Ex 5 Non invertible matrix

$$A := \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \qquad [\lambda \; M] := MEigen(A) = \begin{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \end{bmatrix}$$

Fails for eivenvectors.

Dot numerics values

$$\text{dn\_LinAlgEigenvalues}(A) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\text{dn\_LinAlgEigenvectors}(A) = \begin{bmatrix} 1 & -1 \\ 0 & 2.2204 \cdot 10^{-16} \end{bmatrix}$$

Alvaro