Utils

```
Equrep(e#, a#, b#) := str2num(concat("equrep(", num2str(e#), ",", num2str(a#), ",", num2str(b#), ")
```

```
Find(x#, a#) := n# := [1..length(x#)]
                if (f# := findrows(eval(augment(x#_n#, n#)), a#, 1)) = 0
                  0
                else
                  eval(col(f#, 2))
```

RKA

```
RKA(DE,X,IC',tf,N) solves the Differential Equation system DE
given IC' guess values for the derivatives in X
```

```
RKA(Ev#, Yv#, I'#, TF#, N#) :=
:= [Clear(u#, v#, t#) ky# := 0 ke# := 0 kn# := 0 kv# := 0]
   [ts# := 0 ys# := 0 U# := 0 V# := 0 E# := 0 M# := 0 IC# := eval(matrix(0, 3))]
   for s# ∈ eval(num2str(sys2mat_1(Yv#)))
     [a# b#] := [findstr(s#, "(") findstr(s#, ")")]
     [ts# := substr(s#, a# + 1, b# - a# - 1) "Assuming all equals"]
     ys#_{ky# := ky# + 1} := substr(s#, 1, a# - 1)
   for s# ∈ eval(num2str(sys2mat_1(Ev#)))
     g# := strsplit(s#, "≡")
     if findstr(s#, concat("(", ts#, ")")) = -1
       [a# b#] := [findstr(s#, "(") findstr(s#, ")")]
       [to# := substr(s#, a# + 1, b# - a# - 1) "Assuming all equals"]
       ico# := substr(s#, 1, a# - 1)
       S# := strrep(strrep(ico#, "'", ""), concat("(", ts#, ")"), "")
       IC# := stack(IC#, [Find(ys#, S#)_1 rows(strsplit(ico#, "'"))^{g#_2}])
     else
       if length(g#) = 1
         E#_{ke# := ke# + 1} := str2num(s#)
       else
         E#_{ke# := ke# + 1} := str2num(g#_1) - str2num(g#_2)
   ic# := augment(1000 · col(IC#, 1) + col(IC#, 2), col(IC#, 3))
   ic# := eval(col(csort(ic#, 1), 2))
   for n# ∈ [1..ky#]
     M#_{n#} := eval(max(col(findrows(IC#, n#, 1), 2)))
     [kn# := kn# + 1 a# := concat(ys#_{n#}, "(", ts#, ")")]
     E# := Equrep(E#, a#, u#_{kn#})
     for m# := 1, m# ≤ M#_{n#}, m# := m# + 1
       a# := strrep(a#, "(", "'(")
       if m# ≤ M#_{n#} - 1
         [ V#  := u#  ]
```

```
    kn# := kn# + 1  kv# := kv# + 1  "v" kv# := "u" kn#
    E# := Equrep(E#, a#, u# kn#)
  kv# := kv# + 1  U# n# := v# n#  V# kv# := v# n#
  E# := Equrep(E#, a#, v# n#)
"From here is solve then Rkadapt"
RK_F#(v#) := eval(E#)
RK_D#(t#, u#) := str2num(concat(ts#, ":", num2str(t#)))
                 v# := al_nleqsolve(I'#, RK_F#)
                 eval(V#)
Rkadapt(str2num(ic#), str2num(to#), TF#, N#, RK_D#)
```

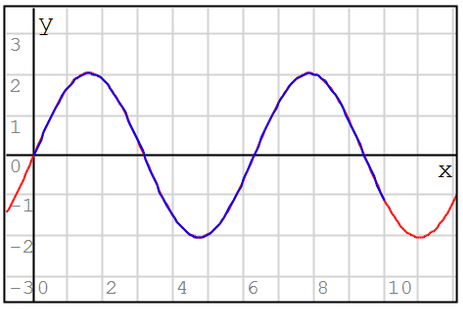
Example

DE System

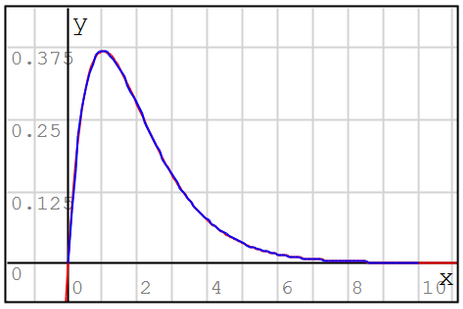
$$DE := \begin{cases} y''(t)\cdot t - x(t)\cdot y(t) = -\dfrac{t\cdot(2 - t + 2\cdot\sin(t))}{e^{t}} \\ x''(t) - \cos(4\cdot t)\cdot y'(t) = -\dfrac{2\cdot\sin(t)\cdot e^{t} + \cos(4\cdot t)\cdot(1 - t)}{e^{t}} \\ x'(0) = 2 \\ y(0) = 0 \\ y'(0) = 1 \\ x(0) = 0 \end{cases}$$

$$RK := RKA\left(DE, \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}', \begin{bmatrix} 0 \\ -2 \end{bmatrix}, 10, 100\right)$$

$$\begin{cases} xs(t) := 2\cdot\sin(t) \\ ys(t) := t\cdot e^{-t} \end{cases}$$



$$\begin{cases} \text{augment}(\text{col}(RK, 1), \text{col}(RK, 2)) \\ xs(x) \end{cases}$$



$$\begin{cases} \text{augment}(\text{col}(RK, 1), \text{col}(RK, 4)) \\ ys(x) \end{cases}$$

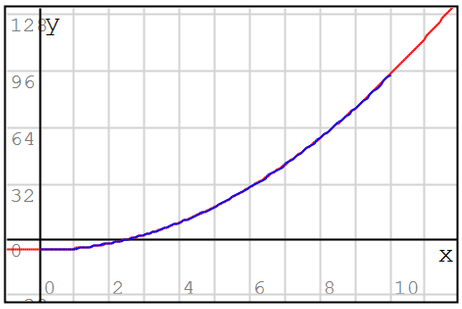
Example

DE System

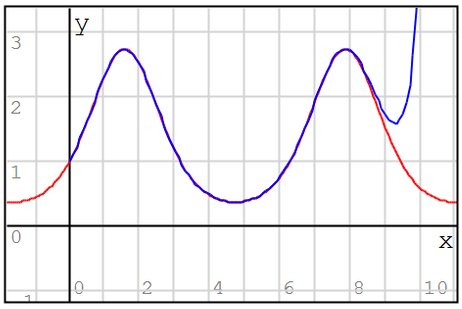
$$DE := \begin{cases} a''(t)\cdot b(t) - c(t) = \dfrac{4\cdot\left(1 + e^{\sin(t)}\right) - t^{2}}{2} \\ b''(t) - t\cdot c'(t) = e^{\sin(t)}\cdot\left((\cos(t))^{2} - \sin(t)\right) - t^{2} \\ c''(t)\cdot a''(t) - 3\cdot b(t) = 2 - 3\cdot e^{\sin(t)} \\ a'(0) = 0 \\ b(0) = 1 \\ b'(0) = 1 \\ c'(0) = 0 \\ a(0) = -6 \\ c(0) = -2 \end{cases}$$

$$RK := RKA\left(DE, \begin{cases} a(t) \\ b(t), \\ c(t) \end{cases} \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}, 10, 100\right)$$

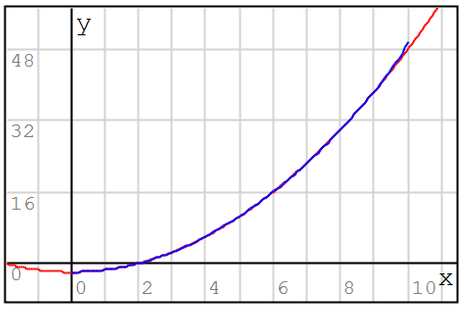
$$\begin{cases} xs(t) := t^2 - 6 \\ ys(t) := e^{\sin(t)} \\ zs(t) := 0.5 \cdot t^2 - 2 \end{cases}$$



$$\begin{cases} \text{augment}(\text{col}(RK, 1), \text{col}(RK, 2)) \\ xs(x) \end{cases}$$



$$\begin{cases} \text{augment}(\text{col}(RK, 1), \text{col}(RK, 4)) \\ ys(x) \end{cases}$$



$$\begin{cases} \text{augment}(\text{col}(RK, 1), \text{col}(RK, 6)) \\ zs(x) \end{cases}$$

Example

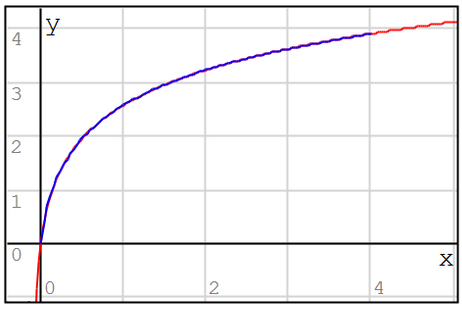
DE System

$$\alpha := 12$$

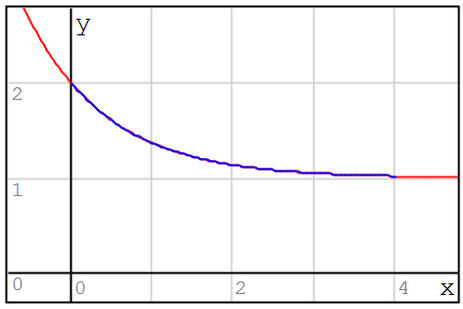
$$DE := \begin{cases} v''(\sigma) \cdot v(\sigma) - \sigma \cdot u(\sigma) = \dfrac{e^{\sigma} + 1 - \sigma \cdot \ln(1 + \alpha \cdot \sigma) \cdot e^{2 \cdot \sigma}}{e^{2 \cdot \sigma}} \\ u'(\sigma) \cdot v''(\sigma) + \cos(2 \cdot \sigma) = \dfrac{\alpha + \cos(2 \cdot \sigma) \cdot (1 + \alpha \cdot \sigma) \cdot e^{\sigma}}{(1 + \alpha \cdot \sigma) \cdot e^{\sigma}} \\ v(0) = 2 \\ v'(0) = -1 \\ u(0) = 0 \end{cases}$$

$$RK := RKA\left(DE, \begin{cases} u(\sigma) \\ v(\sigma)' \end{cases} \begin{bmatrix} \alpha \\ 1 \end{bmatrix}, 4, 100\right)$$

$$\begin{cases} us(\sigma) := \ln(1 + \alpha \cdot \sigma) \\ vs(\sigma) := e^{-\sigma} + 1 \end{cases}$$



$$\begin{cases} \text{augment}(\text{col}(RK, 1), \text{col}(RK, 2)) \\ us(x) \end{cases}$$



$$\begin{cases} \text{augment}(\text{col}(RK, 1), \text{col}(RK, 3)) \\ vs(x) \end{cases}$$

Alvaro