

spice# and MNA

- Numerical Inverse Laplace Transform
- MNA
- Spice
- MNA Method

MNA Method

Given a linear circuit with n Nodes and m sources, solve the system $\mathbf{A} \cdot \mathbf{X} = \mathbf{B}$

$$\mathbf{A}_{n+m, n+m} = \begin{bmatrix} G_{n \cdot n} & C_{n \cdot m} \\ C_{m \cdot n} & D_{m \cdot m} \end{bmatrix} \quad \mathbf{X}_{n+m} = \begin{bmatrix} V(N\#) \\ I(M\#) \end{bmatrix}$$

$$\mathbf{B}_{n+m} = \begin{bmatrix} \Sigma(I_X) \\ V_m \end{bmatrix}$$

G have the passive elements: the diagonal elements $G(n,n)$ are the sum of the conductance of each element connected to the node n , and the others $G(i,j)$ are the negative conductance of the element connected to the nodes i,j .

C have the voltage sources, where if a voltage source is connected to nodes m, n , then $C(m,n) = \pm 1$, or zero otherwise.

D is the same of B , but with dependent sources.

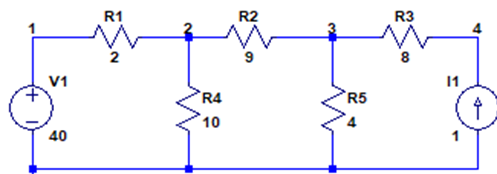
X have the node voltages and currents through voltage sources.

For b , $b(n) =$ sum of the currents through the passive elements into the node n , and $b(m) =$ value of the independent voltage sources m .

Example MNA Method

Vb	1	0
R1	1	2
R2	2	3
R3	3	4
R4	2	0
R5	3	0
Is	0	4

Vb	1	0	40
R1	1	2	2
R2	2	3	9
R3	3	4	8
R4	2	0	10
R5	3	0	4
Is	0	4	1



$$G1 := \frac{1}{R1}$$

$$G2 := \frac{1}{R2}$$

$$G3 := \frac{1}{R3}$$

$$G := \begin{bmatrix} G1 & -G1 & 0 & 0 \\ -G1 & G1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & G2 & -G2 & 0 \\ 0 & -G2 & G2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & G3 & -G3 \\ 0 & 0 & -G3 & G3 \end{bmatrix}$$

$$G4 := \frac{1}{R4}$$

$$G5 := \frac{1}{R5}$$

$$G := G + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & G4 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & G5 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad D = [0]$$

$$C = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} V_b \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -I_s \\ V_b \end{bmatrix} \quad \begin{array}{l} \text{at node 4} \\ \text{First V source} \end{array}$$

$$[A \ B \ X \ V \ I] := MNA("Xs")$$

$$X = \begin{bmatrix} V(1) \\ V(2) \\ V(3) \\ V(4) \\ I(Vb) \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ I_s \\ V_b \end{bmatrix}$$

$$A = \begin{bmatrix} \frac{1}{R1} & -\frac{1}{R1} & 0 & 0 & 1 \\ -\frac{1}{R1} & \frac{(R2 + R4) \cdot R1 + R2 \cdot R4}{R1 \cdot R2 \cdot R4} & -\frac{1}{R2} & 0 & 0 \\ 0 & -\frac{1}{R2} & \frac{(R3 + R5) \cdot R2 + R3 \cdot R5}{R2 \cdot R3 \cdot R5} & -\frac{1}{R3} & 0 \\ 0 & 0 & -\frac{1}{R3} & \frac{1}{R3} & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\text{Clear}(D, G, G1, G2, G3, G4, G5) = 1$$

Numerically:

$$[A \ B \ X \ V \ I] := MNA(\chi)$$

$$V = \begin{bmatrix} 40 \\ 30 \\ 12 \\ 20 \end{bmatrix} \quad A^{-1} \cdot B = \begin{bmatrix} 40 \\ 30 \\ 12 \\ 20 \\ -5 \end{bmatrix}$$

$$I = [-5]$$

Vb	1	0	40
R1	1	2	2
R2	2	3	9
R3	3	4	8
R4	2	0	10
R5	3	0	4
Is	0	4	1

$\chi := "X"$

$$\text{Spice} \left(\chi, ".OP", \begin{bmatrix} V(1) \\ V(2) \\ V(3) \\ V(4) \\ I(Vb) \end{bmatrix} \right) = \begin{bmatrix} 0 & 40 \\ 0 & 30 \\ 0 & 12 \\ 0 & 20 \\ 0 & -5 \end{bmatrix} \quad X = \begin{bmatrix} V(1) \\ V(2) \\ V(3) \\ V(4) \\ I(Vb) \end{bmatrix}$$

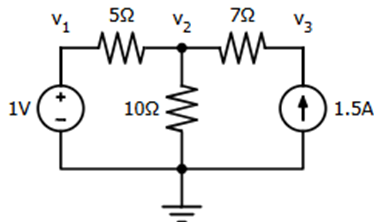
Example spice# MNA Method

spice sharp also uses MNA. From:

https://spicesharp.github.io/SpiceSharp/articles/tutorials/writing_behaviors/mo

V1	1	0	1
R1	1	2	5
R2	2	3	7
R3	2	0	10
I1	0	3	1.5

$\chi := "X"$



$$\begin{pmatrix} \frac{1}{5\Omega} & -\frac{1}{5\Omega} & 0 & 1 \\ -\frac{1}{5\Omega} & \frac{1}{5\Omega} + \frac{1}{10\Omega} + \frac{1}{7\Omega} & -\frac{1}{7\Omega} & 0 \\ 0 & -\frac{1}{7\Omega} & \frac{1}{7\Omega} & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ i_V \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1.5A \\ 1V \end{pmatrix}$$

$$[A \ B \ X \ V \ I] := MNA(\chi)$$

$$A = \begin{bmatrix} 0.2 & -0.2 & 0 & 1 \\ -0.2 & 0.44 & -0.14 & 0 \\ 0 & -0.14 & 0.14 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 1.5 \\ 1 \end{bmatrix} \quad X = \begin{bmatrix} V(1) \\ V(2) \\ V(3) \\ I(V1) \end{bmatrix}$$

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$$V = \begin{bmatrix} 1 \\ 5.67 \\ 16.17 \end{bmatrix} \quad \text{Spice}(\chi, ".OP", X) = \begin{bmatrix} 0 & 1 \\ 0 & 5.67 \\ 0 & 16.17 \\ 0 & 0.93 \end{bmatrix}$$

$$I = [0.93]$$

☐ — Ohm's Law

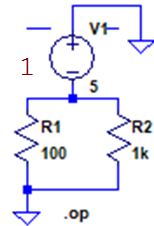
Ohm's Law

V1	1	0
R1	1	0
R2	1	0

$\chi_S := "\chi_S"$

V1	1	0	5
R1	1	0	100
R2	1	0	1k

$\chi := "\chi"$



$$[A \ B \ X \ V \ I] := MNA(\chi_S) \quad V = [V1] \quad I = \left[-\frac{(R1 + R2) \cdot V1}{R1 \cdot R2} \right]$$

$$[A \ B \ X \ V \ I] := MNA(\chi) \quad V = [5] \quad I = [-0.06]$$

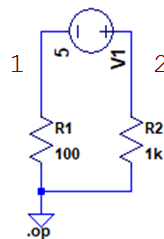
$$\text{Spice}(\chi, ".OP", I(V1)) = [0 \ -0.06]$$

V1	1	2
R1	1	0
R2	2	0

$\chi_S := "\chi_S"$

V1	1	2	5
R1	1	0	100
R2	2	0	1k

$\chi := "\chi"$



$$[A \ B \ X \ V \ I] := MNA(\chi_S) \quad V = \begin{bmatrix} \frac{R1 \cdot V1}{R1 + R2} \\ -\frac{R2 \cdot V1}{R1 + R2} \end{bmatrix} \quad I = \left[-\frac{V1}{R1 + R2} \right]$$

$$[A \ B \ X \ V \ I] := MNA(\chi) \quad V = \begin{bmatrix} 0.45 \\ -4.55 \end{bmatrix}$$

$$I = [0]$$

$$\text{Spice} \left(\text{"X"}, \text{"OP"}, \begin{bmatrix} V(1) \\ V(2) \\ I(V1) \end{bmatrix} \right) = \begin{bmatrix} 0 & 0.45 \\ 0 & -4.55 \\ 0 & 0 \end{bmatrix}$$

☐—Resistive networks

Resistive networks

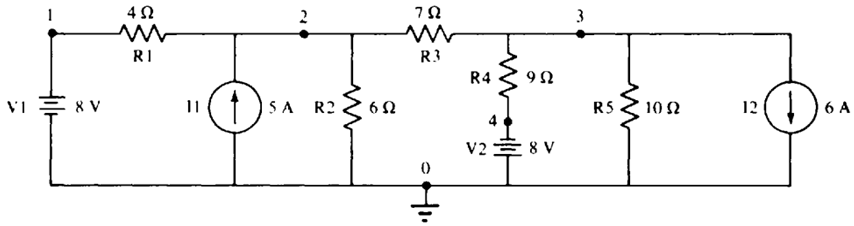


Fig. 7-1

```
V1 1 0 8
R1 1 2 4
I1 0 2 5
R2 2 0 6
R3 2 3 7
R4 3 4 9
V2 0 4 8
R5 3 0 10
I2 3 0 6
```

$$[A \ B \ X \ V \ I] := MNA(X)$$

$$VI := \text{Spice}(X, \text{"OP"}, X)$$

$$X := \text{"X"}$$

$$V = \begin{bmatrix} 8 \\ 8.41 \\ -16.07 \\ -8 \end{bmatrix}$$

$$VI = \begin{bmatrix} 0 & 8 \\ 0 & 8.41 \\ 0 & -16.07 \\ 0 & -8 \\ 0 & 0.1 \\ 0 & 0.9 \end{bmatrix}$$

$$X = \begin{bmatrix} V(1) \\ V(2) \\ V(3) \\ V(4) \\ I(V1) \\ I(V2) \end{bmatrix}$$

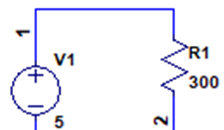
☐—RL Circuit

RL Circuit

Symbolic and Numerical analysis

```
V1 1 0 V.1
R1 1 2 R.1
```

```
V1 1 0 5
R1 1 2 300
```

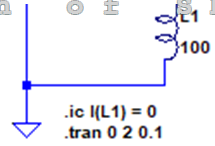


`L120` `g.1`

`L1` `0=100`

`χS := "χS"`

`χ := "χ"`



`O := [".IC V(2) = 0"
".TRAN 0 3 0.01"]`

`[A B X V I] := MNA (χS)`

inverting laplace transforms:

$$V = \begin{bmatrix} V_1 \\ \frac{V_1 \cdot s \cdot L_1}{R_1 + s \cdot L_1} \end{bmatrix}$$

$$I = \begin{bmatrix} -\frac{V_1}{R_1 + s \cdot L_1} \end{bmatrix}$$

$$\mathcal{L}^{-1} \left(\frac{V_2}{s} \right) = V_2(t)$$

$$\mathcal{L}^{-1} \left(-\frac{I_1}{s} \right) = I_L(t)$$

$$V_2(t) := V_1 \cdot e^{-\frac{R_1 \cdot t}{L_1}}$$

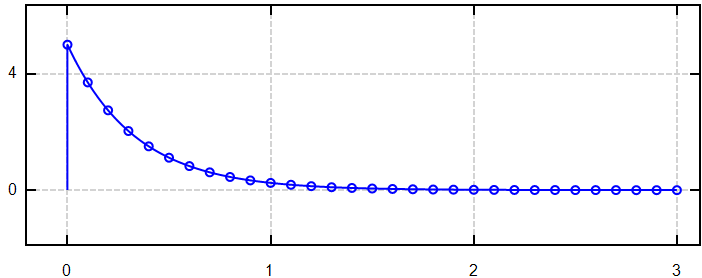
$$I_L(t) := \frac{V_1}{R_1} \cdot \left(1 - e^{-\frac{R_1 \cdot t}{L_1}} \right)$$

`V1 := 5`

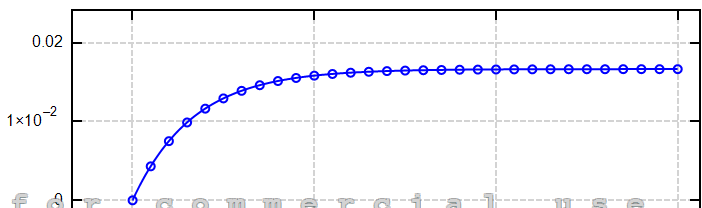
`R1 := 300`

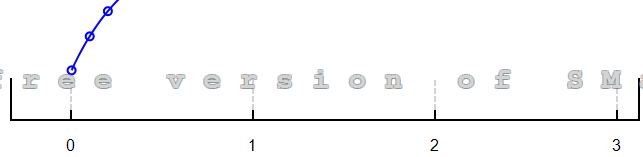
`L1 := 100`

`T := [0, 0.1..3]`



`{ Spice (χ, O, V(2))
augment (T, V2(T)) }`





$$\begin{cases} \text{Spice}(\chi, 0, I(L1)) \\ \text{augment}(T, I_L(T)) \end{cases}$$

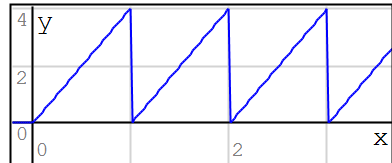
Pulse

```
V1 1 0 PULSE(0 4 0 1-1n 0 1n 1)
R1 1 2 300
L1 2 0 100
```

Clear(V_1)=1

$\chi := "x"$

$$\begin{cases} V_1(t) := 4 \cdot (t - u(t-1)) \cdot t \\ u(t) := 0.5 \cdot (1 + \text{sign}(t)) \end{cases}$$



$$\{V_1(\text{mod}(x, 1)) \cdot (x > 0)\}$$

Using

$$\mathcal{L}(u(t-a) \cdot t) = e^{-a \cdot s} \cdot \mathcal{L}(t+a)$$

2nd shift prop

$$\mathcal{L}(u(t-a)) = \frac{e^{-a \cdot s}}{s}$$

$$\mathcal{L}(V_1(t)) = 4 \cdot \mathcal{L}(t) - 4 \cdot e^{-a \cdot s} \cdot \mathcal{L}(t+1)$$

then

$$\mathcal{L}_{V_1} := - \frac{4 \cdot (-1 + e^{-s} \cdot s + e^{-s})}{s^2}$$

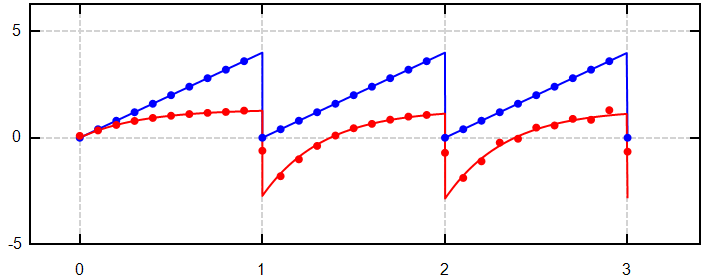
in the freq domain

Finally, using that a periodic function f with period p and "first period" expression f^* have

$$\mathcal{L}(f(t)) = \frac{\mathcal{L}(f^*(t))}{1 - e^{-p \cdot s}}$$

$$\mathcal{L}_{V_2} := \frac{\text{equirep}(V_2, V_1, \mathcal{L}_{V_1})}{1 - e^{-1 \cdot s}} = - \frac{4 \cdot (s - e^s + 1)}{s \cdot (3 + s) \cdot (e^s - 1)}$$

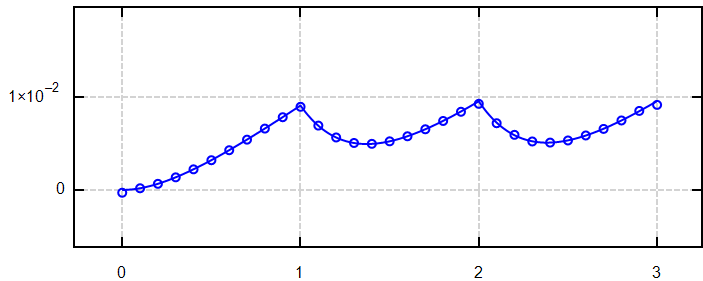
$$\mathcal{L}I_1 := \frac{1 - e^{-1 \cdot s}}{1 - e^{-1 \cdot s}} = - \frac{s - e^s + 1}{25 \cdot s^2 \cdot (3 + s) \cdot (e^s - 1)}$$



```

{
  Spice(x, 0, V(1))
  Spice(x, 0, V(2))
  augment(T, V_1(mod(T, 1)), ".")
  augment(F(LV_2, T), ".")
}

```



```

{
  Spice(x, 0, I(L1))
  augment(F(LI_1, T), ".")
}

```

RC Circuit

RC Circuit

Symbolic and Numerical analysis

```

V1 1 0
R1 1 2
C1 2 0

```

```

V1 1 0 12
R1 1 2 22k
C1 2 0 0.9m

```

$\chi^s := "X^s"$
Not

$\chi := "X"$

for commercial use

$$[A \ B \ X \ V \ I] := MNA(\chi s)$$

$$I = \left[-\frac{s \cdot C1 \cdot V1}{1 + s \cdot C1 \cdot R1} \right]$$

$$V = \left[\begin{array}{c} V1 \\ \frac{V1}{1 + s \cdot C1 \cdot R1} \end{array} \right]$$

$$[A \ B \ X \ V \ I] := MNA(\chi)$$

$$V_2(t) := \text{maple} \left(\text{invlaplace} \left(\frac{V_2}{s}, s, t \right) \right) = 12 \cdot \left(1 - \exp \left(-\frac{5 \cdot t}{99} \right) \right)$$

$$I_C(t) := \text{maple} \left(\text{invlaplace} \left(-\frac{I_1}{s}, s, t \right) \right) = \frac{3 \cdot \exp \left(-\frac{5 \cdot t}{99} \right)}{5500}$$

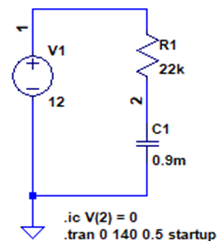
RL Circuit

Symbolic and
Numerical analysis

```
V1 1 0 12
R1 1 2 22k
C1 2 0 0.9m
```

$\chi := "X"$

```
O := [ ".IC V(2) = 0"
      ".TRAN 0 140 5" ]
```



$$[A \ B \ X \ V \ I] := MNA(\chi s)$$

inverting laplace transforms:

$$V = \left[\begin{array}{c} V1 \\ \frac{V1}{1 + s \cdot C1 \cdot R1} \end{array} \right]$$

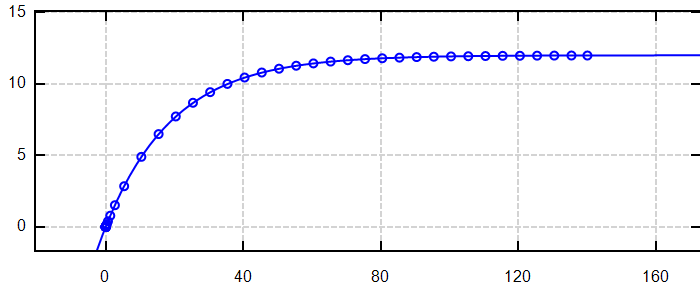
$$I = \left[-\frac{s \cdot C1 \cdot V1}{1 + s \cdot C1 \cdot R1} \right]$$

$$\mathcal{L} \left(\frac{V_2}{s} \right) = V_2(t)$$

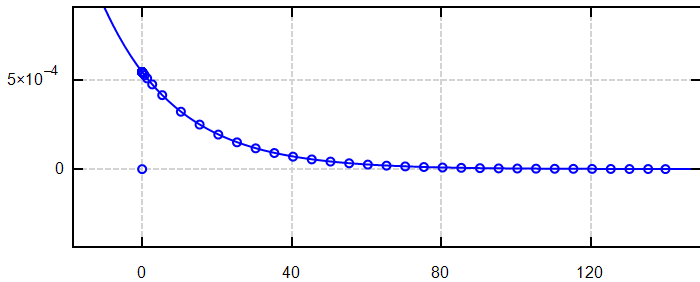
$$\mathcal{L} \left(-\frac{I_1}{s} \right) = I_L(t)$$

$$V_2(t) := 12 \cdot \left(1 - e^{-\frac{5 \cdot t}{99}} \right)$$

$$I_L(t) := \frac{3}{5500} \cdot e^{-\frac{5 \cdot t}{99}}$$



$$\begin{cases} v_2(t) \\ \text{Spice}(\chi, 0, V(2)) \end{cases}$$



$$\begin{cases} I_C(t) \\ \text{Spice}(\chi, 0, I(C1)) \end{cases}$$

— Controlled sources

— E: VCVS

Voltage Controlled Voltage Source (VCVS)

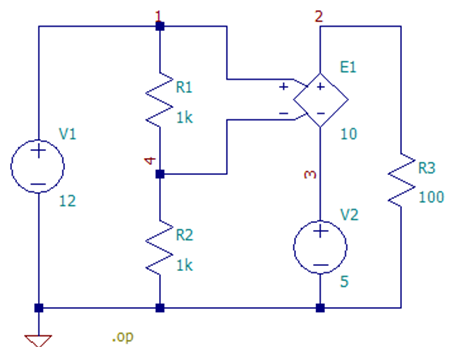
Example

```
V1 1 0 12
V2 3 0 5
R1 1 4 1k
R2 0 4 1k
R3 2 0 100
E1 2 3 1 4 10
```

$\chi := "X"$

$[A \ B \ X \ V \ I] := MNA(\chi)$

$VI := \text{Spice}(\chi, ".OP", X)$

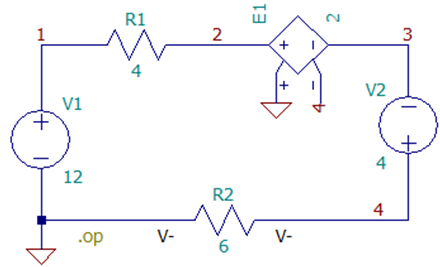


$$V = \begin{bmatrix} 65 \\ 5 \\ 6 \end{bmatrix} \quad VI = \begin{bmatrix} 0 & 65 \\ 0 & 5 \\ 0 & 6 \\ 0 & -0.006 \\ 0 & -0.65 \\ 0 & -0.65 \end{bmatrix} \quad X = \begin{bmatrix} V(2) \\ V(3) \\ V(4) \\ I(V1) \\ I(V2) \\ I(E1) \end{bmatrix}$$

Example

R1	2	1	4
R2	4	0	6
V1	1	0	12
V2	4	3	4
E1	2	3	0 4 2

$\chi := "X"$



$[A \ B \ X \ V \ I] := MNA(\chi)$

$VI := Spice(\chi, ".OP", X)$

$$V = \begin{bmatrix} 12 \\ 44 \\ -52 \\ -48 \end{bmatrix} \quad VI = \begin{bmatrix} 0 & 12 \\ 0 & 44 \\ 0 & -52 \\ 0 & -48 \\ 0 & 8 \\ 0 & 8 \\ 0 & -8 \end{bmatrix} \quad X = \begin{bmatrix} V(1) \\ V(2) \\ V(3) \\ V(4) \\ I(V1) \\ I(V2) \\ I(E1) \end{bmatrix}$$

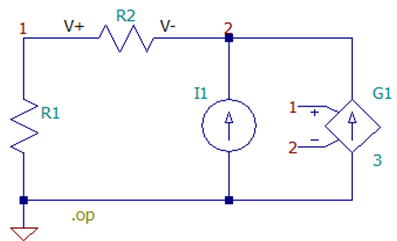
☐—G: CCCS

Current Controlled Current Source (CCCS)

Example

R1	1	0	200
R2	2	1	100
I1	0	2	4
G1	0	2	1 2 3

$\chi := "X"$



$[A \ B \ X \ V \ I] := MNA(\chi)$

$VI := Spice(\chi, ".OP", X)$

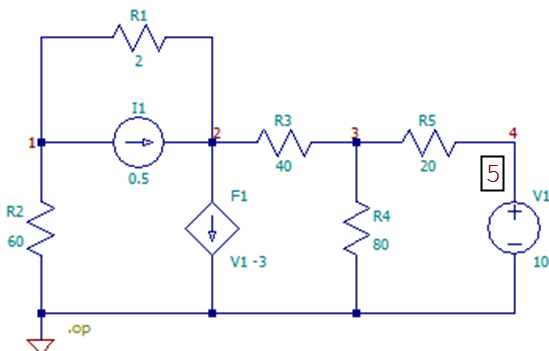
$$V = \begin{bmatrix} 2.66 \\ 3.99 \end{bmatrix} \quad VI = \begin{bmatrix} 0 & 2.66 \\ 0 & 3.99 \\ 0 & -3.99 \end{bmatrix} \quad X = \begin{bmatrix} V(1) \\ V(2) \\ I(G1) \end{bmatrix}$$

☐ F: CCCS

Current Controlled Current Source (CCCS)

Example

Adding a node for the MNA method:



I1	1	2	0.5
V1	5	0	10
R1	2	1	2
R2	0	1	60
R3	2	3	40
R4	3	0	80
R5	4	3	20
F1	2	0	4 5 -3

$X := "X"$

I1	1	2	0.5
V1	4	0	10
R1	2	1	2
R2	0	1	60
R3	2	3	40
R4	3	0	80
R5	4	3	20
F1	2	0	V1 -3

$X_S := "X.S"$

$$[A \ B \ X \ V \ I] := MNA(X)$$

$$VI := Spice(X_S, ".OP", X)$$

$$V = \begin{bmatrix} 51.82 \\ 54.55 \\ 21.3 \\ 10 \\ 10 \end{bmatrix}$$

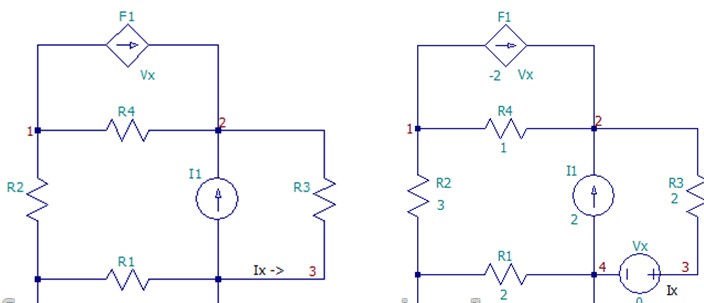
$$I = \begin{bmatrix} 0.56 \\ -1.69 \end{bmatrix}$$

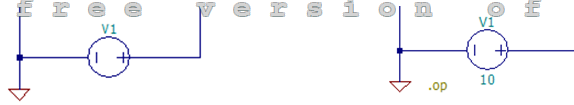
$$VI = \begin{bmatrix} 0 & 51.82 \\ 0 & 54.55 \\ 0 & 21.3 \\ 0 & 10 \\ "V(5)" & "-" \\ 0 & 0.56 \\ 0 & -1.69 \end{bmatrix}$$

$$X = \begin{bmatrix} V(1) \\ V(2) \\ V(3) \\ V(4) \\ V(5) \\ I(V1) \\ I(F1) \end{bmatrix}$$

Example

Adding 0V source:





```
I1 4 2 2
V1 4 0 10
R1 4 0 2
R2 0 1 3
R3 3 2 2
R4 1 2 1
F1 1 2 3 4 -2
```

$\chi := "\chi"$

```
I1 4 2 2
V1 4 0 10
R1 4 0 2
R2 0 1 3
R3 3 2 2
R4 1 2 1
Vx 3 4 0
F1 1 2 Vx -2
```

$\chi_S := "\chi.S"$

$$[A \ B \ X \ V \ I] := MNA(\chi)$$

$$VI := Spice(\chi_S, ".OP", X)$$

$$V = \begin{bmatrix} 6.75 \\ 9.5 \\ 10 \\ 10 \end{bmatrix}$$

$$I = \begin{bmatrix} -7.25 \\ 0.5 \end{bmatrix}$$

$$VI = \begin{bmatrix} 0 & 6.75 \\ 0 & 9.5 \\ 0 & 10 \\ 0 & 10 \\ 0 & -7.25 \\ 0 & 0.5 \end{bmatrix}$$

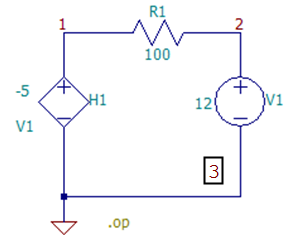
$$X = \begin{bmatrix} V(1) \\ V(2) \\ V(3) \\ V(4) \\ I(V1) \\ I(F1) \end{bmatrix}$$

□—H: CCVS

Current Controlled Voltage Source (CCVS)

Example

For MNA we need to add N3 for specify the current-sensing branch.



```
V1 2 3 {V1}
R1 1 2 {R1}
H1 1 0 3 0 {G}
```

$\chi := "\chi"$

```
V1 2 0 {V1}
R1 1 2 {R1}
H1 1 0 V1 {G}
```

$\chi_2 := "\chi_2"$

```
.PARAM V1=12 R1=100 G=-5
.OP
```

$\Pi := "\Pi"$

$$[A \ B \ X \ V \ I] := MNA(\chi, \Pi) \quad VI := Spice(\chi_2, \Pi, X)$$

$$V = \begin{bmatrix} 0.57 \\ 12 \\ 0 \end{bmatrix}$$

$$VI = \begin{bmatrix} 0 & 0.5714 \\ 0 & 12 \\ 0 & - \end{bmatrix}$$

$$X = \begin{bmatrix} V(1) \\ V(2) \\ V(3) \end{bmatrix}$$

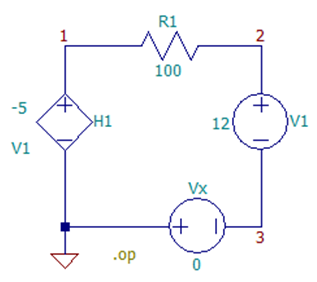
Not for commercial use

$$I = \begin{bmatrix} -0.11 \\ -0.11 \\ 0.11 \end{bmatrix} \quad \forall I = \begin{bmatrix} 0 & -0.1143 \\ \text{"Ix (H1)"} & \text{"-"} \\ 0 & 0.1143 \end{bmatrix} \quad X = \begin{bmatrix} I(V1) \\ Ix(H1) \\ I(H1) \end{bmatrix}$$

Another way for spice is adding a 0 V source in series with the current-sensing branch.

```
V1 2 3 {V1}
R1 1 2 {R1}
Vx 3 0 0
H1 1 0 Vx {G}
```

Renaming:
 $X_5 = Ix(H1)$
 $X_5 := I(Vx)$

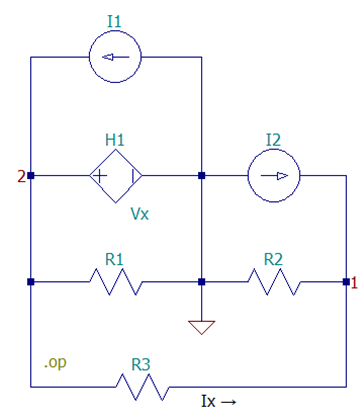


$\chi3 := \text{"}\chi3\text{"}$

$$Spice(\chi3, \Pi, X) = \begin{bmatrix} 0 & 0.57 \\ 0 & 12 \\ 0 & 0 \\ 0 & -0.11 \\ 0 & -0.11 \\ 0 & 0.11 \end{bmatrix} \quad X = \begin{bmatrix} V(1) \\ V(2) \\ V(3) \\ I(V1) \\ I(Vx) \\ I(H1) \end{bmatrix}$$

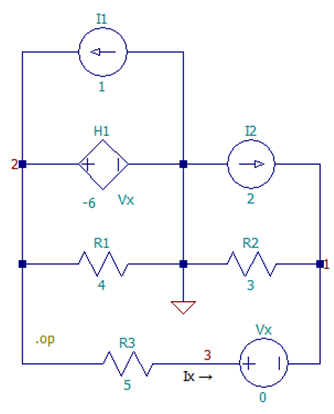
That is optional when we have a V source, but we need it if the current control is not across a V, like in the next example.

Example



Adding 0V source:

$V_1 = V_3$



```
I1 0 2 1
I2 0 1 2
R1 0 2 4
R2 0 1 3
```

```
I1 0 2 1
I2 0 1 2
R1 0 2 4
R2 0 1 3
```

$[A B X V I] := MNA(X)$
 $X_4 := I(Vx)$

```
R3=2 3 5
H1 2 0 3 1 -6
```

$\chi := "X"$

```
R3=2 3 5
Vx 3 1 0
H1 2 0 Vx -6
```

$\chi_S := "X.S"$

$VI := Spice(\chi_S, ".OP", X)$

$$V = \begin{bmatrix} 4.71 \\ 2.57 \\ 4.71 \end{bmatrix}$$

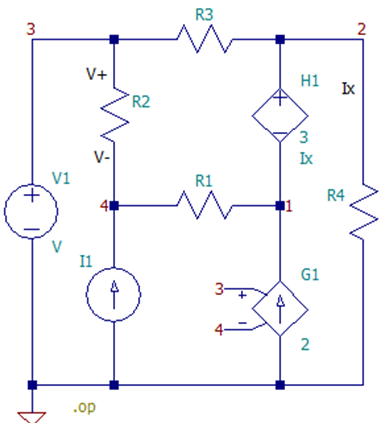
$$I = \begin{bmatrix} -0.43 \\ 0.79 \end{bmatrix}$$

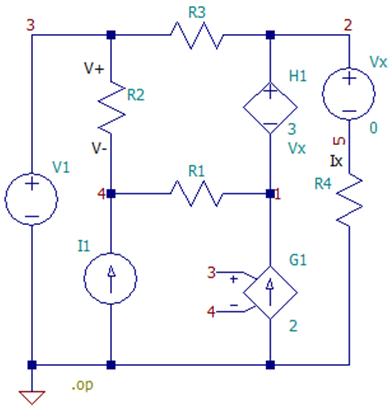
$$VI = \begin{bmatrix} 0 & 4.71 \\ 0 & 2.57 \\ 0 & 4.71 \\ 0 & -0.43 \\ 0 & 0.79 \end{bmatrix}$$

$$X = \begin{bmatrix} V(1) \\ V(2) \\ V(3) \\ I(Vx) \\ I(H1) \end{bmatrix}$$

Controlled sources Example

Example





```
I1 0 4 0.6
V1 3 0 5
R1 1 4 10
R2 3 4 20
R3 2 3 10
R4 0 5 20
H1 2 1 2 5 3
G1 0 1 3 4 2
```

$\chi := "X"$

```
I1 0 4 0.6
V1 3 0 5
R1 1 4 10
R2 3 4 20
R3 2 3 10
R4 0 5 20
Vx 2 5 0
H1 2 1 Vx 3
G1 0 1 3 4 2
```

$\chi_S := "X.S"$

$[A B X V I] := MNA(\chi)$

Renaming

$X_7 = Ix(H1) \quad X_7 := I(Vx)$

$VI := Spice(\chi_S, ".OP", X)$

$$V = \begin{bmatrix} -0.17 \\ -0.2 \\ 5 \end{bmatrix}$$

$$VI = \begin{bmatrix} 0 & -0.17 \\ 0 & -0.2 \\ 0 & 5 \end{bmatrix}$$

$$X = \begin{bmatrix} V(1) \\ V(2) \\ V(3) \end{bmatrix}$$

$$I = \begin{bmatrix} -0.2 \\ -0.49 \\ -0.01 \\ 0.53 \\ -1.1 \end{bmatrix} \quad VI = \begin{bmatrix} 0 & -0.2 \\ 0 & -0.49 \\ 0 & -0.01 \\ 0 & 0.53 \\ 0 & -1.1 \end{bmatrix} \quad X = \begin{bmatrix} V(4) \\ V(5) \\ I(V1) \\ I(Vx) \\ I(H1) \\ I(G1) \end{bmatrix}$$

□ opamps

Opamp for spice#

```
.subckt OA 1 2 3
G1 0 2 1 3 100k
R3 2 0 1
C3 2 0 0.001591549430919
.ends OA
```

spice# $_Aol := 100 \cdot 10^3$ $_GBW := 10 \cdot 10^6$
 subcircuit $\frac{_Aol}{2 \cdot \pi \cdot _GBW} = 0.001591549430919$

$$OA := \begin{bmatrix} "O" & 3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ "I" \end{bmatrix}$$

OA# := description(OA)
 Ideal opamp stamp

Summary of basic op amp circuits

Shorthands:

$Vs(n) := \left| \begin{array}{c} MNA("X") \\ 4 \\ n \end{array} \right|$ Symbolic MNA

$Vn(n) := \left| \begin{array}{c} MNA("X", "II") \\ 4 \\ n \end{array} \right|$ Numeric MNA

$V\#(n) := \left| \begin{array}{c} Spice \left("X", \begin{bmatrix} ".OP" \\ "II" \\ OA\# \end{bmatrix}, V(n) \right) \\ \end{array} \right|$ Numeric spice#

```
.PARAM Vi=5 R1=3K R2=2K R3=6K
.PARAM V1=8 V2=5 V3=12 Rf=2K
```

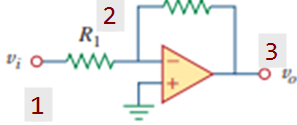
Numeric values

$\Pi := 1$

Inverter amplifier

```
Vi 1 0 {Vi}
R1 1 2 {R1}
R2 2 3 {R2}
```

$Vs(3) = -\frac{Vi \cdot R2}{R1}$



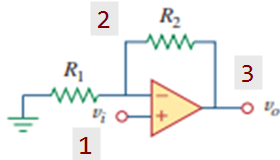
X1 0 3 2 OA

$\chi := 1$

$$V_n(3) = -3.33$$

$$V\#(3) = [0 \ -3.33]$$

Non-inverter amplifier



Vi 1 0 {Vi}
R1 0 2 {R1}
R2 2 3 {R2}
X1 1 3 2 OA

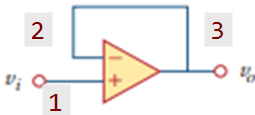
$\chi := 1$

$$V_s(3) = \frac{(R1 + R2) \cdot Vi}{R1}$$

$$V_n(3) = 8.33333$$

$$V\#(3) = [0 \ 8.3332]$$

Voltage Follower



Vi 1 0 {Vi}
X1 1 2 2 OA

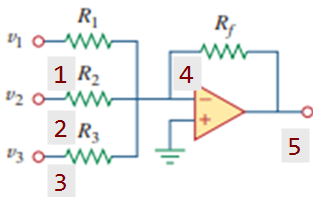
$\chi := 1$

$$V_s(2) = Vi$$

$$V_n(2) = 5$$

$$V\#(2) = [0 \ 5]$$

Summer



V1 1 0 {V1}
V2 2 0 {V2}
V3 3 0 {V3}
R1 1 4 {R1}
R2 2 4 {R2}
R3 3 4 {R3}
Rf 4 5 {Rf}
X1 0 5 4 OA

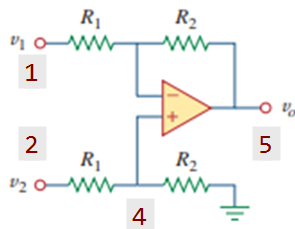
$\chi := 1$

$$V_n(5) = -14.3333$$

$$V\#(5) = [0 \ -14.3329]$$

$$V_s(5) = -\frac{R_f \cdot ((V1 \cdot R2 + V2 \cdot R1) \cdot R3 + V3 \cdot R2 \cdot R1)}{R3 \cdot R2 \cdot R1}$$

Difference Amplifier



V1 1 0 {V1}
V2 2 0 {V2}
R1 1 3 {R1}
R2 2 4 {R2}
R3 3 4 {R3}
Rf 4 5 {Rf}
X1 0 5 4 OA

$\chi := 1$

$$V_n(5) = -6.7778$$

$$V\#(5) = [0 \ -6.7776]$$

$$V_s(5) = -\frac{R_f \cdot (V1 \cdot R2 + (R1 + R3) \cdot V2)}{(R1 + R3) \cdot R2}$$

□ opamp: Capacitance multiplier

Capacitance multiplier

```
Vb 1 0 V.b
R1 1 3 1k
R2 1 2 100k
C1 2 0 100n
X1 2 3 3 OA
```

```
Vb 1 0 PULSE(0 12 0 0 0 5m 10m)
R1 1 3 1k
R2 1 2 100k
C1 2 0 100n
X1 2 3 3 OA
```

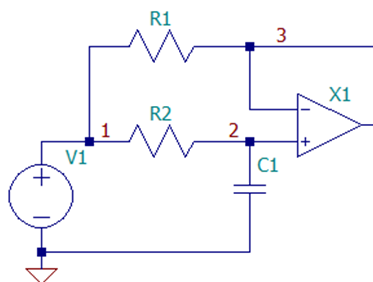
$\chi := "X"$

$\chi_S := "X.S"$

```
O := [ OA#
      ".TRAN 0 60m 0.1m" ]
```

$T := [0, 1..60] \cdot 10^{-3}$ rows(T) = 61

```
[ A B X V I ] := MNA( $\chi$ )
```

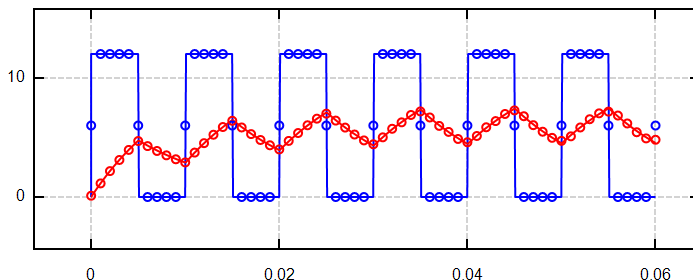


Laplace transform of the pulse:

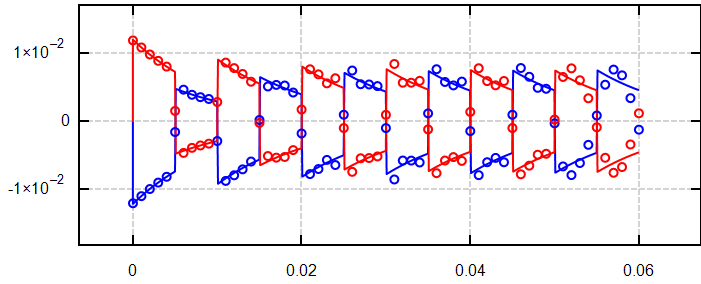
$T_o := 5 \cdot 10^{-3}$ $u(x) := 0.5 \cdot (1 + \text{sign}(x))$

$V_b(t) := 12 \cdot (u(t) - u(t - T_o))$ Time domain

$V_b := \frac{12}{s \cdot (1 + e^{-T_o \cdot s})}$ Freq domain



```
[ Spice( $\chi_S$ , 0, V(1))
  Spice( $\chi_S$ , 0, V(2))
  augment( T, V_b( T - 2 \cdot T_o \cdot \text{trunc}( \frac{T}{2 \cdot T_o} ) ) )
  FL(V_2, T) ]
```

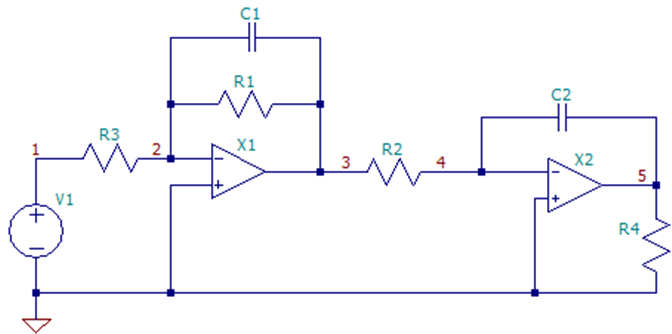


$$\begin{cases} \text{Spice}(\chi_s, 0, I(Vb)) \\ \text{Spice}(\chi_s, 0, I(R1)) \\ \mathcal{FL}(I_1, T) \\ \mathcal{FL}(I_2, T) \end{cases}$$

Clear(Vb) = 1

☐ — opamp: DC OP Point

DC OP Point



Vb	1	0
R1	2	3
R2	3	4
R3	1	2
R4	5	0
C1	2	3
C2	4	5
X1	0	2 3 OA
X2	0	4 5 OA

Vb	1	0	12
R1	2	3	1k
R2	3	4	100k
R3	1	2	100k
R4	5	0	100k
C1	2	3	10n
C2	4	5	10n
X1	0	2 3	OA
X2	0	4 5	OA

$$[R1 \ R2 \ R3 \ R4] := \left[\frac{1}{G1} \ \frac{1}{G2} \ \frac{1}{G3} \ \frac{1}{G4} \right]$$

$$[A \ B \ X] := MNA(\chi_s, "", "ABX")$$

$$A \cdot X = B$$

$\chi^s := "$ χs $"$

$\chi := "$ χ $"$

$$\begin{bmatrix} G3 & -G3 & 0 & 0 & 0 & 1 & 0 & 0 \\ -G3 & G3 + G1 + s \cdot C1 & -(G1 + s \cdot C1) & 0 & 0 & 0 & 1 & 0 \\ \text{Not} & \text{for} & \text{commercial} & \text{use} & & & & \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & -(G1 + s \cdot C1) & G2 + G1 + s \cdot C1 & -G2 & 0 & 0 & 0 \\ 0 & 0 & -G2 & G2 + s \cdot C2 & -s \cdot C2 & 0 & 0 & 1 \\ 0 & 0 & 0 & -s \cdot C2 & G4 + s \cdot C2 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \end{bmatrix}$$

$$X = \begin{bmatrix} V(1) \\ V(2) \\ V(3) \\ V(4) \\ V(5) \\ I(Vb) \\ I(X1) \\ I(X2) \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ Vb \\ 0 \\ 0 \end{bmatrix} \quad [A \ B \ X] := MNA(\chi, "", "ABX")$$

$$A(s) := A \quad B(s) := B$$

rref(augment(A(0), B(0))) ToDo: rref!

$$Spice \left(\chi, \begin{bmatrix} ".OP" \\ OA\# \end{bmatrix}, \begin{bmatrix} "V(1)" \\ "V(2)" \\ "V(3)" \\ "I(R4)" \\ "I(Vb)" \end{bmatrix} \right) = \begin{bmatrix} 0 & 12 \\ 0 & 1.21 \cdot 10^{-9} \\ 0 & 1.2 \cdot 10^{-9} \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Clear(s, Vb, R1, R2, R3, R4, C1, C2) = 1

Alvaro appVersion(4) = "1.2.9018.0"