CO₂ FORCING SATURATION STUDY

⊞— Data2

$$Data2=\begin{bmatrix}1850 & 196847600 & 2835458800\\ 1851 & 198778540 & 3018492200\\ 1852 & 207595090 & 3065093600\\ 1853 & 217236980 & 3122834700\\ 1854 & 255038560 & 3169778800\\ 1855 & 260259860 & 3138780700\\ 1856 & 278245630 & 3247129900\\ & \vdots & \end{bmatrix}$$

$Date:=\mathrm{col}(Data2,1)$

$Emission:=\mathrm{col}(Data2,2)\cdot 10^{-9}$

$DATA:=\mathrm{augment}(Date,Emission)$

$n_1:=165$

$N:=\mathrm{rows}(Data2)=174$

$\max(Date)=2023$

$t_0:=\min(Date)=1850$

$t_1:=Date_{n_1}=2014$

FIT TO EMISSION DATA

⊟— FIT EMISSION DATA

SMATH FIT USING al_nelqsolve

$X:=DATA_{[1..N]1}$ $\quad Y:=DATA_{[1..N]2}$ $\quad nselect:=[1,3..N]$ $\quad vline(t,y):=\begin{bmatrix}t & 0\\ t & y\end{bmatrix}$

$$F_{exp}(t,u):=\left|\begin{array}{l}[a\ b\ c]:=[u_1\ u_2\ u_3]\\ \overrightarrow{c+a\cdot e^{b\cdot\frac{t-t_0}{t_0}}}\end{array}\right.$$

$$u_{exp}:=\mathrm{al\_nleqsolve}\left(\begin{bmatrix}10\\10\\-10\end{bmatrix},\varphi(u):=F_{exp}(X,u)-Y\right)=\begin{bmatrix}1.4923\\36.1473\\-2.1976\end{bmatrix}$$

TRYING TO CREATE A LINEAR FIT WHERE THE DATA LEVELS OFF BETWEEN 2014 AND 2023

$XX:=DATA_{[n_1..\mathrm{rows}(DATA)]1}$ $\quad YY:=DATA_{[n_1..\mathrm{rows}(DATA)]2}$

$$F_{lin}(t,u):=\left|\begin{array}{l}[a\ b]:=[u_1\ u_2]\\ \overrightarrow{a+b\cdot\frac{t-t_1}{t_1}}\end{array}\right.$$

$$u_{lin}:=\mathrm{al\_nleqsolve}\left(\begin{bmatrix}10\\10\end{bmatrix},\varphi(u):=F_{lin}(XX,u)-YY\right)=\begin{bmatrix}35.2403\\487.6924\end{bmatrix}$$

setprop("XYPlot'XAxis'Min", 1850) = 1

setprop("XYPlot'XAxis'Max", 2050) = 1

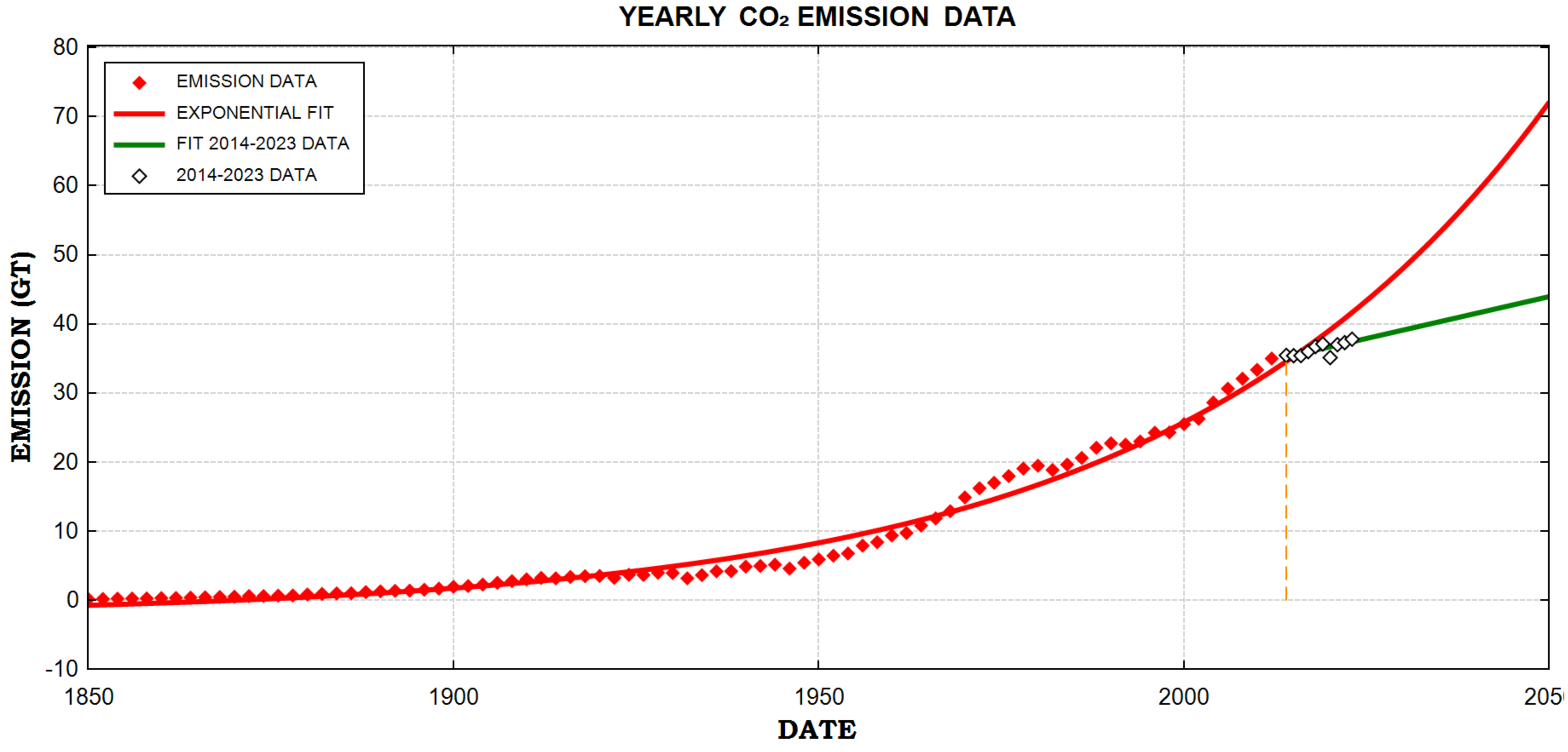
setprop("XYPlot'YAxis'Min", −10) = 1

setprop("XYPlot'YAxis'Max", 100) = 1

setprop("XYPlot'XAxis'Tick", 50) = 1

setprop("XYPlot'YAxis'Tick", 10) = 1

$$Plot:=\left\{\begin{array}{l}\mathrm{augment}\left(X_{nselect},Y_{nselect}\right)\\ F_{exp}\left(t,u_{exp}\right)\\ \left\{\begin{array}{ll}F_{lin}\left(t,u_{lin}\right) & \text{if } t>t_1\\ \text{""} & \text{otherwise}\end{array}\right.\\ \mathrm{augment}(XX,YY)\\ vline\left(t_1,F_{lin}\left(t_1,u_{lin}\right)\right)\end{array}\right.$$



TO GET CUMULATIVE EMISSIONS WE MUST SOLVE THE FOLLOWING ODE

$$\frac{d}{d\,t}y(t)=Q(t)-\lambda\cdot y(t)$$

$$Q(t):=\left\{\begin{array}{ll}F_{exp}\left(t,u_{exp}\right) & \text{if } t\le t_1\\ F_{lin}\left(t,u_{lin}\right) & \text{otherwise}\end{array}\right.$$

Where Q(t) is the driving function = yearly emissions and λ = decay of CO₂ in the atmosphere.

SOLUTION TO THE ODE TO GET CUMULATIVE EMISSIONS

⊟— ODE TO GET TOTAL CONCENTRATION

ODE TO GET TOTAL CONCENTRATION

$\tau := 300$ $\lambda := \frac{1}{\tau}$ $t_{end} := 2200$ $ppm := \frac{1}{7.8}$ $\tau := \left[t_0 .. t_{end} \right]$ $\text{rows}(\tau) = 351$

$$CO_2(t) := ppm \cdot \mathbf{e}^{-\lambda \cdot t} \cdot \int_{t_0}^{t} Q(s) \cdot \mathbf{e}^{\lambda \cdot s} \, \mathrm{d}s$$

or

$$\begin{cases} y'(t) = ppm \cdot Q(t) - \lambda \cdot y(t) \\ y(t_0) = ppm \cdot Q(t_0) \end{cases}$$

$$Plot := \begin{cases} CO_2 \\ vline(t_1, CO_2(t_1)) \end{cases}$$

or

using time as number of steps in rkadapt:

$$CO_2 := \text{Rkadapt}(y(t), t_{end}, \text{rows}(\tau) + 1)$$

$$Plot := \begin{cases} CO_2(t) \\ vline(t_1, CO_2(t_1)) \end{cases}$$

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setprop("CO2'XAxis'Min", 1850) = 1      setprop("CO2'XAxis'Tick", 50) = 1
setprop("CO2'XAxis'Max", 2250) = 1
setprop("CO2'YAxis'Min", -50) = 1       setprop("CO2'YAxis'Tick", 200) = 1
setprop("CO2'YAxis'Max", 1400) = 1
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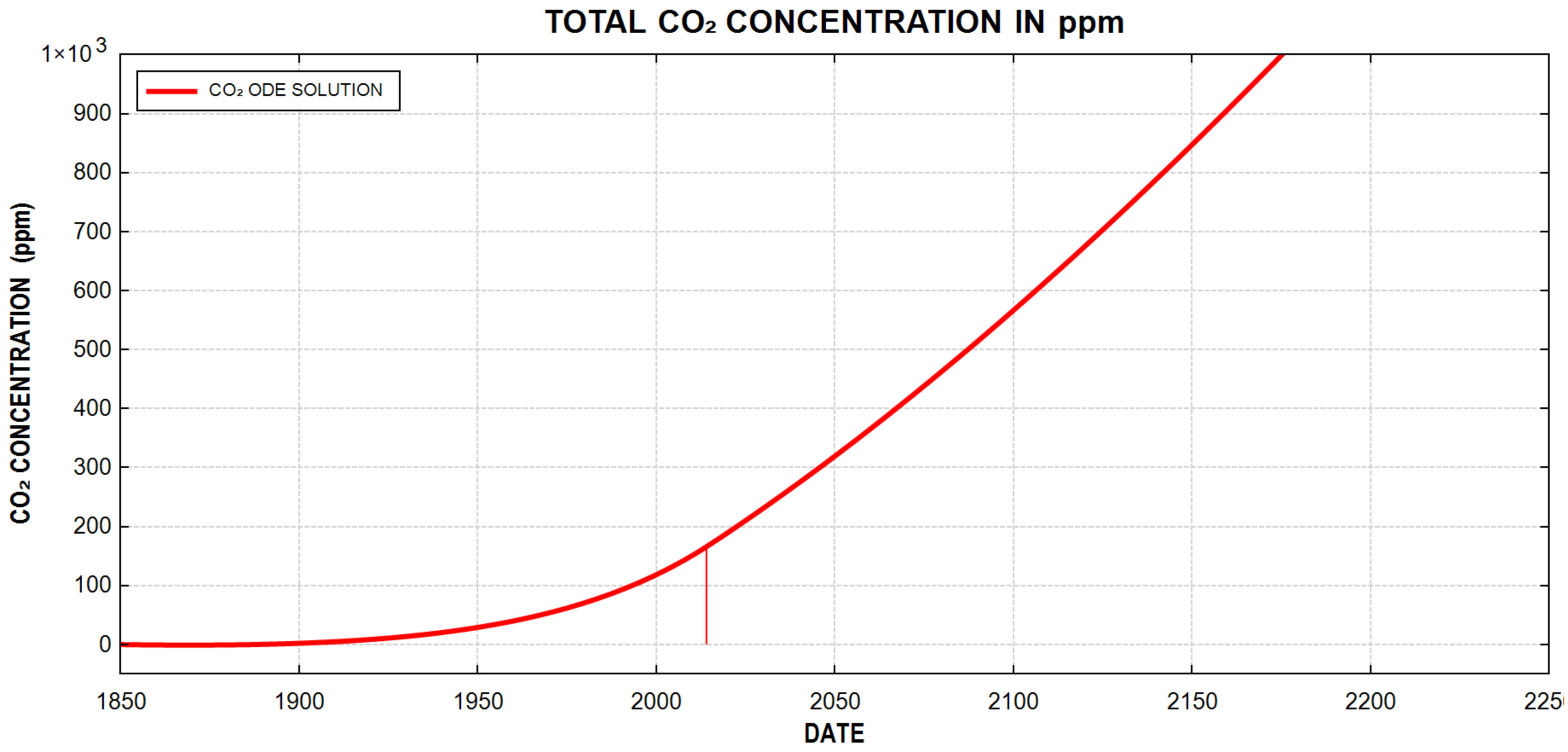


Why???

There is a discrepancy between your method and mine, and I have not an idea why:

$$\begin{aligned} &\text{for } k \in [1 .. \text{length}(\tau)] \\ &\quad \text{if } k \le n_1 \\ &\qquad Y_k := ppm \cdot \mathbf{e}^{-\lambda \cdot \tau_k} \cdot \int_{t_0}^{\tau_k} F_{exp}(s, u_{exp}) \cdot \mathbf{e}^{\lambda \cdot s} \, \mathrm{d}s \\ &\quad \text{else} \\ &\qquad Y_k := ppm \cdot \mathbf{e}^{-\lambda \cdot \tau_k} \cdot \int_{t_1}^{\tau_k} F_{lin}(s, u_{lin}) \cdot \mathbf{e}^{\lambda \cdot s} \, \mathrm{d}s + Y_{n_1} \end{aligned}$$

Your max:

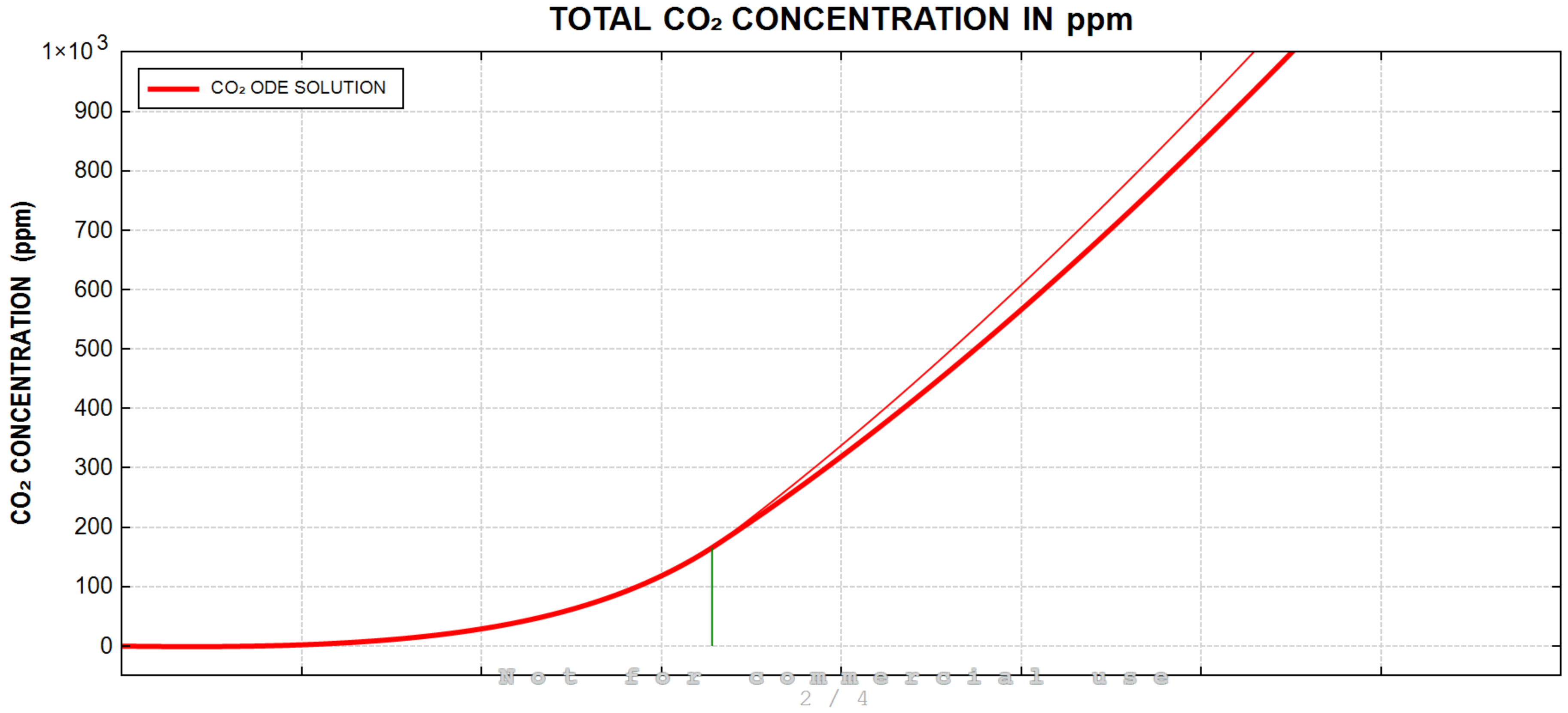
$\max(Y) = 1233.2524$

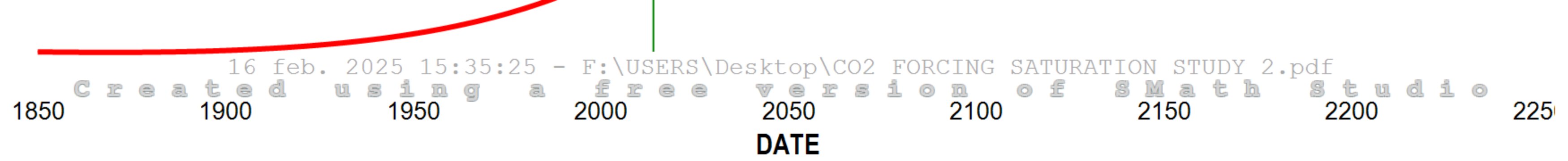
$Y_{n_1} = 165.679$

$$Plot := \begin{cases} CO_2 \\ \text{augment}(\tau, Y) \\ vline(t_1, CO_2(t_1)) \end{cases}$$

My max: By integration: $\boxed{CO_2(t_{end}) = 1156.6978}$ By ODE solver: $\boxed{\max(\text{col}(CO_2, 2)) = 1156.6686}$ $y(t_{end}) = 1156.6686$

$CO_2(t_1) = 165.679$ $y(t_1) = 165.6248$ (y function is generated by Rkadapt function)





ODE TO GET TOTAL CONCENTRATION

NORMALIZE ODE SOLUTION TO ACTUAL PPM DATA

$PPMDATA := \mathrm{importData}_{XLSX}(\text{"WORLD CO2 ppm UPDATE.xlsx"}, \text{"co2-long-term-concentration"}, \text{"B2:C131"})$

WORLD PPM DATA

$PPMDATA_{130\,2} = 419.315$

$PPMDATA_{130\,1} = 2023$

$CO_{2\;174\,1} = 2022.017$

$CO_{2\;174\,2} = 197.9841$

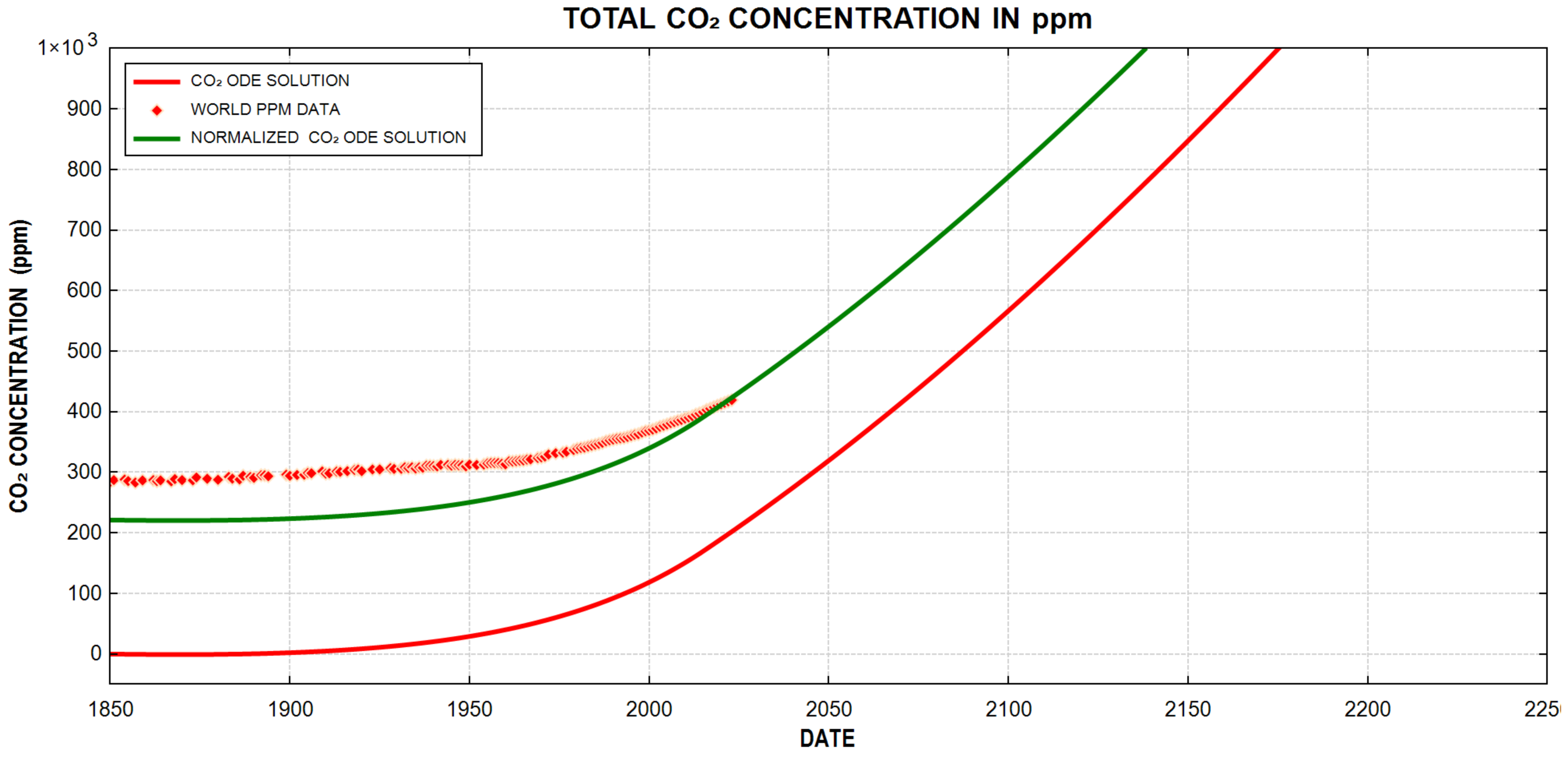
$NORM := PPMDATA_{130\,2} - CO_{2\;174\,2}$

$CO_{2NORM} := \mathrm{col}(CO_2, 2) + NORM$

$CO_{2NORM} := \mathrm{augment}(\mathrm{col}(CO_2, 1), CO_{2NORM})$

$\mathrm{rows}(PPMDATA) = 130$

$$Plot := \begin{cases} CO_2 \\ PPMDATA \\ CO_{2NORM} \end{cases}$$



PHYSICAL CONSTANTS

EARTH ALBEDO: $\alpha := 0.32956$

SOLAR CONSTANT: $S := 1370\ \frac{W}{m^2}$

SPECIFIC HEAT CAPACITY OF EARTH: $C_e := 2.08 \cdot 10^8\ \frac{J}{m^2\,K}$

Stephen Boltzmann Constant per degree Kelvin: $\sigma := 5.6704 \cdot 10^{-8}\ \frac{W}{m^2\,K^4}$

Earth Emissivity, since earth is not a black body: $\varepsilon := 0.6062$

TDFC TEMPERATURE ANOMALY

$CO_{2INITIAL} := CO_{2NORM\;1\,2} = 221.2404$

$k := [1 .. \mathrm{rows}(\tau)]$

$$\Delta FCO2_k := 5.35 \cdot \ln\left(\frac{CO_{2NORM\;k\,2} + CO_{2INITIAL}}{CO_{2INITIAL}}\right) \frac{W}{m^2}$$

$T_o := 14.0\ {}^\circ C = 287.15\ K$

$\delta t := 1\ yr$

$T_1 := T_o$

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for κ ∈ [2..(rows(τ))]
    $T_\kappa := \mathrm{eval}\left(\frac{1}{C_e} \cdot \left(\frac{(1-\alpha) \cdot S}{4} + \Delta FCO2_{\kappa-1} - \left(\varepsilon \cdot \sigma \cdot T_{\kappa-1}^{4}\right)\right) \cdot \delta t + T_{\kappa-1}\right)$
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$Temission := \mathrm{augment}(\tau, T - T_o)$

$$Plot := \begin{cases} Temission \\ \mathrm{stack}\left(\left[t_1\ \ 0\right], \mathrm{findrows}(Temission, t_1, 1)\right) \end{cases}$$

setprop("TEMP'XAxis'Min", 1850) = 1

setprop("TEMP'XAxis'Max", 2250) = 1

setprop("TEMP'YAxis'Min", −.5) = 1

setprop("CO2'XAxis'Tick", 100) = 1

setprop("CO2'YAxis'Tick", 0.1) = 1

$$T - T_o = \begin{bmatrix} 0 \\ -0.056 \\ -0.0846 \\ -0.0992 \\ \vdots \end{bmatrix} \Delta{}^\circ C$$

There is a "change" temp unit, Δ°C = Δ°K

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setprop("TEMP'YAxis'Max",2)=1
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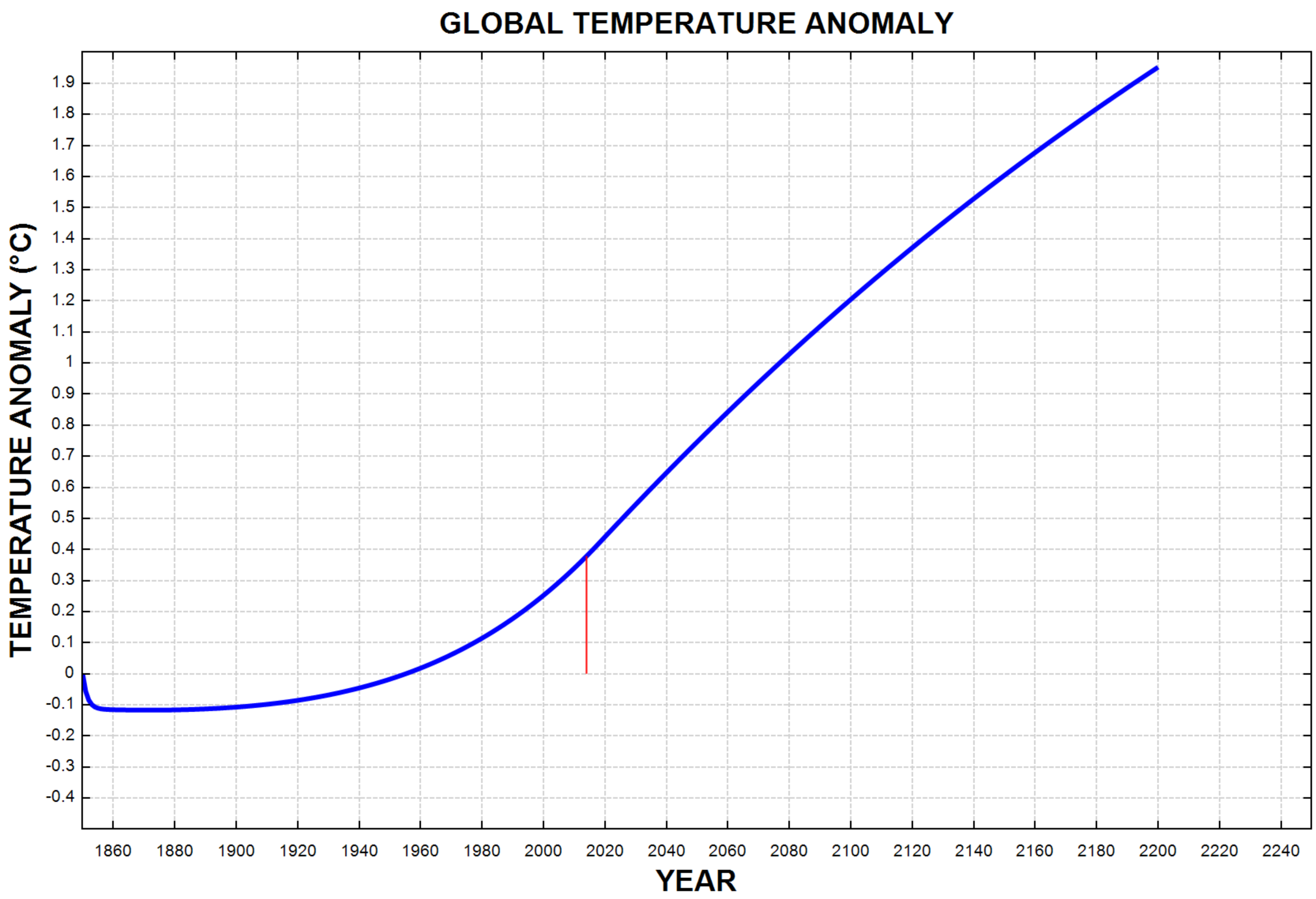

END OF WORKSHEET