$$y(x,a,x0,h) := a \cdot \cosh\left(\frac{x-x0}{a}\right) - a + h$$

$$y'(x,a,x0) := \frac{\partial}{\partial x} y(x,a,x0,h) \rightarrow -\sinh\left(\frac{x0-x}{a}\right)$$

x_1	y_1	L	l	G	g_c	x_2	y_2
(cm)	(cm)	(cm)	(cm)	(gm)	$\left(\frac{gm}{cm}\right)$	(cm)	(cm)
0	10	13	5	20	5	10	17

Without G (for guess values with G)

Начальные приближения

$a1 := 5\ cm \qquad x0 := 1\ cm \qquad h := 10\ cm$

Ограничения

$$y(x_1, a1, x0, h) = y_1 \qquad y(x_2, a1, x0, h) = y_2$$

$$L = \int_{x_1}^{x_2} \sqrt{1 + y'(x, a1, x0)^2}\, dx$$

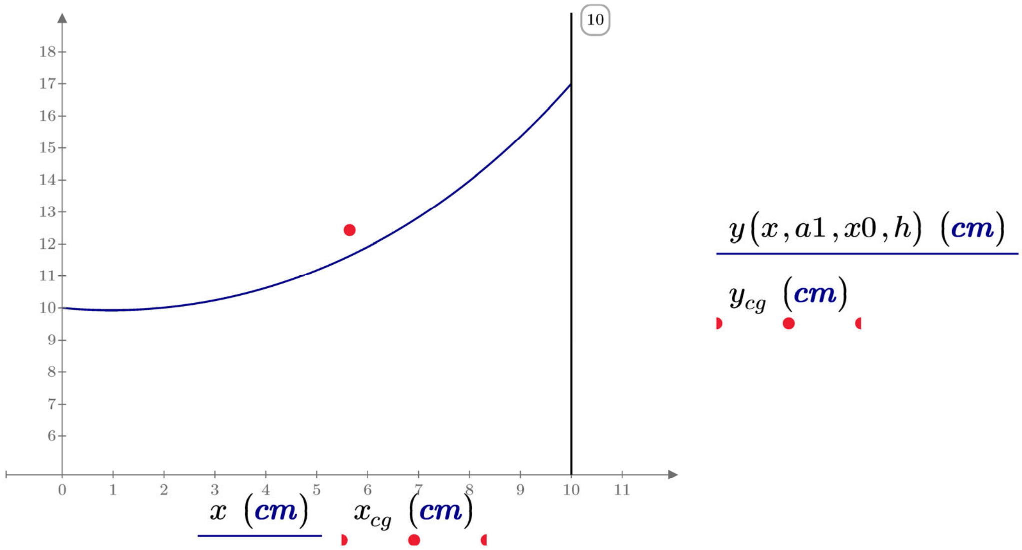
Решатель

$$\begin{bmatrix} a1 \\ x0 \\ h \end{bmatrix} := \mathbf{Find}(a1, x0, \mathbf{h}) = \begin{bmatrix} 6.7 \\ 0.967 \\ 9.93 \end{bmatrix} cm$$

$$x_{cg} := \frac{\int_{x_1}^{x_2} x \cdot \sqrt{1 + y'(x, a1, x0)^2}\, dx}{\int_{x_1}^{x_2} \sqrt{1 + y'(x, a1, x0)^2}\, dx} = 5.646\ cm$$

$$y_{cg} := \frac{\int_{x_1}^{x_2} y(x, a1, x0, h) \cdot \sqrt{1 + y'(x, a1, x0)^2}\, dx}{\int_{x_1}^{x_2} \sqrt{1 + y'(x, a1, x0)^2}\, dx} = 12.431\ cm$$

$$x := x_1, x_1 + \frac{x_2 - x_1}{300} .. x_2$$



With G   Left catenary part + G + Right catenary part

$$PE\left(x_g, a, x0_L, x0_R, h_L, h_R\right) := \mathbf{g} \cdot g_c \cdot \int_{x_1}^{x_g} y\left(x, a, x0_L, h_L\right) \cdot \sqrt{1 + y'\left(x, a, x0_L\right)^2}\, dx + \mathbf{g} \cdot G \cdot y\left(x_g, a, x0_L, h_L\right) + \mathbf{g} \cdot g_c \cdot \int_{x_g}^{x_2} y\left(x, a, x0_R, h_R\right) \cdot \sqrt{1 + y'\left(x, a, x0_R\right)^2}\, dx$$

$x_g := \dfrac{x_2 + x_1}{2} = 5\ \mathbf{cm}$ $a := a1$ $x0_L := x0$ $x0_R := x0$ $h_L := h$ $h_R := h$

$$PE\left(x_g, a, x0_L, x0_R, h_L, h_R\right) = 0.101168\ \mathbf{J}$$

Начальные приближения

Ограничения

$y\left(x_1, a, x0_L, h_L\right) = y_1$ $y\left(x_2, a, x0_R, h_R\right) = y_2$

$$y\left(x_g, a, x0_L, h_L\right) = y\left(x_g, a, x0_R, h_R\right)$$

$l = \int_{x_1}^{x_g} \sqrt{1 + y'\left(x, a, x0_L\right)^2}\, dx$ $L - l = \int_{x_g}^{x_2} \sqrt{1 + y'\left(x, a, x0_R\right)^2}\, dx$

Решатель

$$\begin{bmatrix} x_g \\ a \\ x0_L \\ x0_R \\ h_L \\ h_R \end{bmatrix} := \mathbf{Minimize}\left(PE, x_g, a, x0_L, x0_R, h_L, h_R\right) = \begin{bmatrix} 4.871 \\ 10.257 \\ 0.558 \\ -2.827 \\ 9.985 \\ 7.878 \end{bmatrix} \mathbf{cm}$$

$PE\left(x_g, a, x0_L, x0_R, h_L, h_R\right) = 0.1007369\ J$ $y_g := y\left(x_g, a, x0_R, h_R\right) = 10.905\ cm$

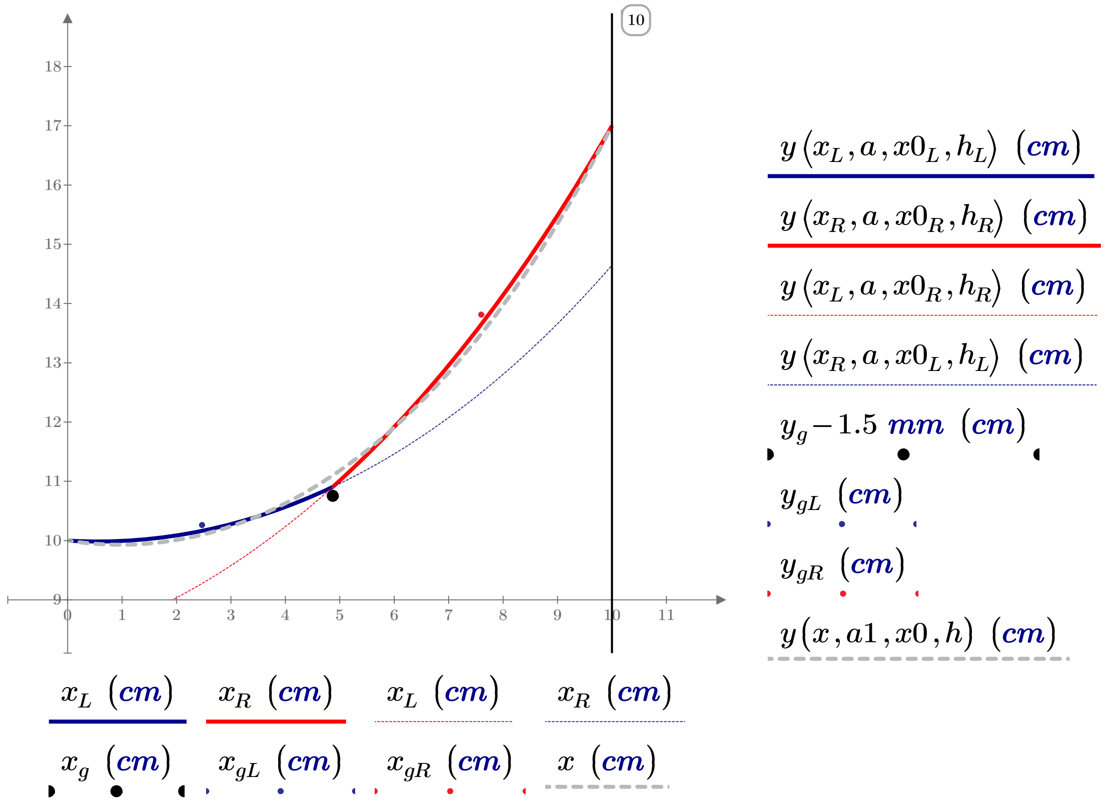
$x_L := x_1, x_1 + \frac{x_g - x_1}{300} .. x_g$ $x_R := x_g, x_g + \frac{x_2 - x_g}{300} .. x_2$

$$x_{gL} := \frac{\int_{x_1}^{x_g} x \cdot \sqrt{1 + y'\left(x, a, x0_L\right)^2}\, dx}{\int_{x_1}^{x_g} \sqrt{1 + y'\left(x, a, x0_L\right)^2}\, dx} = 2.471\ cm$$

$$y_{gL} := \frac{\int_{x_1}^{x_g} y\left(x, a, x0_L, h_L\right) \cdot \sqrt{1 + y'\left(x, a, x0_L\right)^2}\, dx}{\int_{x_1}^{x_g} \sqrt{1 + y'\left(x, a, x0_L\right)^2}\, dx} = 10.263\ cm$$

$$x_{gR} := \frac{\int_{x_g}^{x_2} x \cdot \sqrt{1 + y'\left(x, a, x0_R\right)^2}\, dx}{\int_{x_g}^{x_2} \sqrt{1 + y'\left(x, a, x0_R\right)^2}\, dx} = 7.598\ cm$$

$$y_{gR} := \frac{\int_{x_g}^{x_2} y\left(x, a, x0_R, h_R\right) \cdot \sqrt{1 + y'\left(x, a, x0_R\right)^2}\, dx}{\int_{x_g}^{x_2} \sqrt{1 + y'\left(x, a, x0_R\right)^2}\, dx} = 13.814\ cm$$



$$F_x := a \cdot \boldsymbol{g} \cdot g_c = 0.503\ \boldsymbol{N} \qquad F_{2y} := F_x \cdot y'\left(x_2, a, x0_R\right) = 0.806\ \boldsymbol{N}$$

$$F_2 := \sqrt{{F_x}^2 + {F_{2y}}^2} = 0.95\ \boldsymbol{N}$$

$$F_{1y} := \boldsymbol{g} \cdot L \cdot g_c + \boldsymbol{g} \cdot G - F_{2y} = 0.027\ \boldsymbol{N}$$

$$F_1 := \sqrt{{F_x}^2 + {F_{1y}}^2} = 0.504\ \boldsymbol{N}$$